Magnetoresistance of disordered graphene at high temperatures

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Introduction: magnetotransport of 2D electron gas with quadratic spectrum

Disorder: impurities with the short-range potential:

$$V(\mathbf{r}) = \sum_i u_0 \, \delta(\mathbf{r} - \mathbf{r}_i)$$

The two regimes of magnetoresistance:

1. Classical regime: $\omega_c \tau_q \ll 1$

The Bolzmann equation, the Drude formulas.
ρ_{xx} does not depend on H (at $T=0$), at $T>0$
there are a week dependence $\rho_{xx}(H) \sim H^2$ due to weak dependencies $\tau_q(\varepsilon)$, $m(\varepsilon)$.

2. Quantum regime : $\omega_c \tau_q \sim 1$

Conductivity is to be calculated using Kubo formula and Green functions.
Shubnikov oscillations, quantum Hall effect.

T. Ando and Y. Uemura, Journ. of the Phys. Soc. of Japan 36, 956 (1974).
T. Ando, Journ. of the Phys. Soc. of Japan 37, 1233 (1974).

Formulation of the problem

? : Magnetoresistance of 2D electron gas in graphene at high temperatures (no Shubnikov oscillations). Is there anything peculiar?

$$\hat{H} = s\,\hbar\left(\hat{\boldsymbol{\sigma}}\cdot\left[\hat{\mathbf{k}} + \frac{e}{c\hbar}\mathbf{A}(\mathbf{r})\right]\right) + V(\mathbf{r}) \qquad \varepsilon_{\mathrm{k}} = s\hbar k$$

$$\sigma_{xx} = \int_{-\infty}^{\infty} d\varepsilon \left(-\frac{\partial n_0}{\partial \varepsilon}(\varepsilon)\right)\sigma_{xx}(\varepsilon)$$

In graphene, the cyclotron frequency and the relaxation time strongly depend on electron energy.

$$\omega_{\mathrm{c}} = \frac{eH}{mc} \to \frac{eHs^2}{c\varepsilon} \qquad \frac{1}{\tau_{\mathrm{q}}} = \frac{m\kappa}{\hbar^3} = \frac{2\varepsilon\kappa}{\hbar^3 v_{\mathrm{F}}^2} \to const\,\frac{\varepsilon\kappa}{\hbar^3 s^2} \qquad \varrho^{2\mathrm{D}} = \frac{m}{\pi\hbar^2} \to \frac{N_{\mathrm{deg}}\varepsilon}{2\pi\hbar^2 s^2}$$

$$N_{\mathrm{deg}} = 2\cdot 2 = 4$$

The key magnetotransport parameter $\omega_c\tau_q \sim \varepsilon^{-2}$.

Thus , there are two types of electrons in the sample:

electrons In the classical regime : $\omega_c\tau_q \ll 1$ --- electrons with large energies.

electrons In the quantum regime : $\omega_{\mathrm{c}}\tau_{\mathrm{q}} \gtrsim 1$ --- electrons with small energies.

In it, a nontrivial question arises how do electron states with small or lagre $\omega_c\tau_q$ contribute to magnetoresistance ?

(if the states with small or large value of this parameter are partly populated : $T \gtrsim \varepsilon_{\mathrm{F}}$ *)*

Semiclassical consideration of magnetoconductivity of graphene at high T

$$\frac{1}{\tau_{\rm tr}} = \frac{\kappa\varepsilon}{4\hbar^3 s^2} \equiv \frac{\varepsilon}{\gamma} \qquad \omega_{\rm c}(\varepsilon) = eHs^2/c\varepsilon \qquad m(\varepsilon) = \varepsilon/s^2$$

Drude formula:

$$\sigma_{xx}(\varepsilon) = N_{\rm deg}\frac{e^2 n(\varepsilon)\tau_{\rm tr}(\varepsilon)}{m(\varepsilon)}\frac{1}{1+[\omega_{\rm c}(\varepsilon)\tau_{\rm tr}(\varepsilon)]^2}$$

It turns out that in graphene: $N_{\rm deg}\frac{e^2 n(\varepsilon)\tau_{\rm tr}(\varepsilon)}{m(\varepsilon)} \equiv A$

$$x = \omega_{\rm c}\tau_{\rm tr} \qquad x(\varepsilon) \sim \varepsilon^{-2}$$

We introduce the character marginal energy which separates the quantum and the classical regions on the energy axis: $x(\varepsilon^*) = 1$

$$\varepsilon^* = \sqrt{eH\gamma/c}\, s = \varepsilon_{\rm F}\sqrt{(\omega_{\rm c}\tau_q)|_{\varepsilon_{\rm F}}}$$

$$\varepsilon^* \ll \varepsilon_{\rm F}$$

Semiclassical consideration of magnetoconductivity of grahene at high T

Let us write the energy dependent conductivity in the form:

$$\sigma_{xx}(\varepsilon) = A\{1 + [\tilde{\sigma}^{\mathrm{D}}_{xx}(x) - 1]\} \qquad \tilde{\sigma}^{\mathrm{D}}_{xx}(x) = \frac{\sigma^{\mathrm{D}}_{xx}(x)}{A} = \frac{1}{1+x^2}$$

The term in square brackets goes to zero for $\varepsilon \gg \varepsilon^*$. So, according to $\sigma_{xx} = \int_0^\infty d\varepsilon \left(-\frac{\partial n_0(\varepsilon)}{\partial \varepsilon}\right) \sigma_{xx}(\varepsilon)$ we have the two contributions in magnetoconductivity:

$$\sigma_{xx} = \sigma^{\mathrm{D},0}_{xx} + \Delta\sigma_{xx}$$

$$\sigma^{\mathrm{D},0}_{xx} = A \int_0^\infty d\varepsilon \left(-\frac{\partial n_0(\varepsilon)}{\partial \varepsilon}\right) = A n_0(0) \qquad \Delta\sigma_{xx} = A\varepsilon^* \left(-\frac{\partial n_0(0)}{\partial \varepsilon}\right) I$$

$$\frac{\Delta\sigma_{xx}}{\sigma^{\mathrm{D},0}_{xx}} \sim \sqrt{H}$$

$$I = \int_0^\infty d\tilde{\varepsilon} \, \{\tilde{\sigma}^{\mathrm{D}}_{xx}[x(\tilde{\varepsilon})] - 1\}$$

$\tilde{\varepsilon} = \varepsilon/\varepsilon^*$ $\qquad$ $I = -\pi/2$

But the region " $x \sim 1$" (" $\varepsilon \sim \varepsilon^*$ ") is substantial in the integral for $\Delta\sigma_{xx}$.
Thus, we need quantum consideration.

$$T \gtrsim \varepsilon_{\mathrm{F}}$$

Green function of 2D electrons with quadratic spectrum

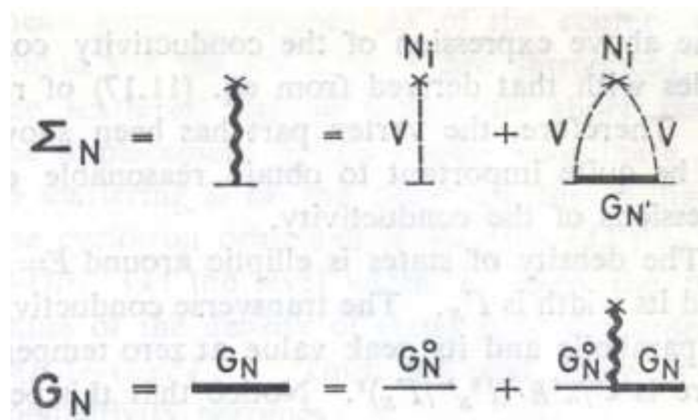
T. Ando and Y. Uemura, Journ. of the Phys. Soc. of Japan 36, 956 (1974).
T. Ando, Journ. of the Phys. Soc. of Japan 37, 1233 (1974).

$$\langle V(\mathbf{r})V(\mathbf{r}')\rangle = \kappa\,\delta(\mathbf{r}-\mathbf{r}') \qquad \kappa = n_{\mathrm{imp}}u_0^2$$

The Basis of Landau levels: $\psi_{N,\alpha}$ $\qquad \varepsilon_N = [N+(1/2)]\hbar\omega_{\mathrm{c}}$

$$G_{N\alpha,N'\alpha',\varepsilon} = G_N(\varepsilon)\delta_{N,N'}\delta_{\alpha,\alpha'}$$

Self-consistent Born approximation:



The Dyson equation takes the form (Σ do not depend on N) :

$$\Sigma(\varepsilon) = \frac{\Gamma(\varepsilon)^2}{4}\sum_{n'}\frac{1}{\varepsilon-\varepsilon_{n'}-\Sigma(\varepsilon)}$$

The Ando's notation: $X(\varepsilon) = \varepsilon - \Sigma(\varepsilon) - (\hbar\omega_{\mathrm{c}}/2)$, $G_N(\varepsilon) = [X(\varepsilon) - N\hbar\omega_{\mathrm{c}}]^{-1}$

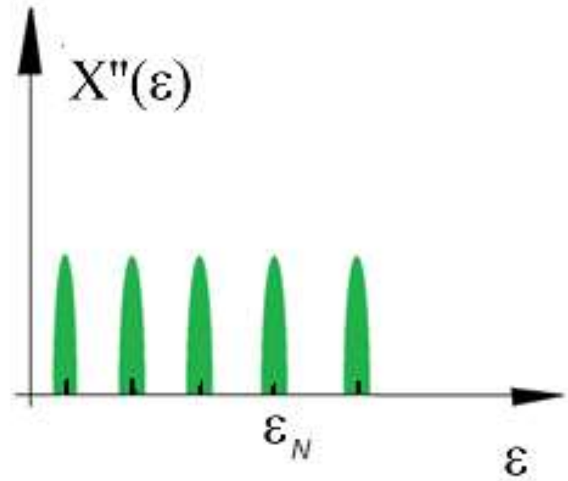
Green function of 2D electrons with quadratic spectrum

1. Weak magnetic fields ($\omega_c \tau_q \ll 1$) : $X'' \approx X''_0 = \frac{\hbar}{2\tau_q}$ $\tau_q(\varepsilon) = \left(\frac{\kappa\,\varepsilon}{\hbar^3 s^2}\right)^{-1}$

2. High magnetic fields ($\omega_c \tau_q \gg 1$) , the solution near some Landau level:

$$X''(\varepsilon) = (\Gamma/2)\sqrt{1 - [(\varepsilon - \varepsilon_N)/\Gamma]^2}.$$

$$X'(\varepsilon) = (\varepsilon - \varepsilon_N)/2 \qquad \Gamma = \frac{2}{\pi}\hbar\sqrt{\frac{\omega_c}{\tau_q}}$$



3. Intermediate magnetic fields: a nontrivial equation:

$$z = \frac{\sinh(\alpha z)}{\cosh(\alpha z) + a}$$

$$X'' = zX''_0$$

$$X''_0 = \hbar/2\tau_q$$

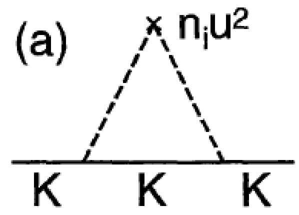
$$\alpha = \pi/\omega_c\tau_q \qquad a = -\cos(2\pi X'/\hbar\omega_c)$$

Green function of 2D electrons in graphene

N. H. Shon and T. Ando, Journ. of the Phys. Soc. of Japan 67, 2421 (1998).

The Landau levels: $\varepsilon_n = \pm\sqrt{2n}\hbar s/l_H$

$$\psi_{n,k_y} = \frac{1}{\sqrt{L}}\frac{1}{\sqrt{2}}e^{-ik_y y}\begin{pmatrix} h_{n-1}(x-x_0) \\ h_n(x-x_0) \end{pmatrix}$$



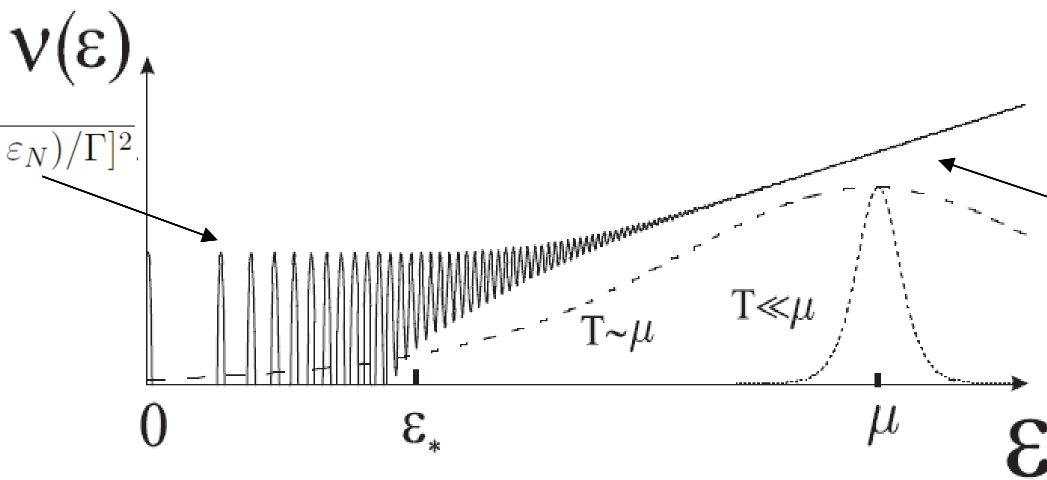
The Green function in coordinate representation is a matrix 2*2;
in (n,k_y) representation it contains mixed terms with $+n$ and $-n$, but for $\varepsilon \gg \hbar\omega_c(\varepsilon)\,,\ \hbar/\tau_q(\varepsilon)$ they are small .

Features, special for graphene:

1. All parameters τ_q, ω_c , m depend on ε. But for short-range disorder : $\Gamma(\varepsilon) = const$
2. we introduce the «local» index $n_0(\varepsilon)$: $\varepsilon - \varepsilon_{n'} = \hbar\omega_c(\varepsilon)[n' - n_0(\varepsilon)]$

The result:

$X''(\varepsilon) = (\Gamma/2)\sqrt{1-[(\varepsilon-\varepsilon_N)/\Gamma]^2}$



$X_0'' = \hbar/2\tau_q(\varepsilon)$

Quantum conductivity of 2D electron gas with quadratic spectrum

T. Ando and Y. Uemura, Journ. of the Phys. Soc. of Japan 36, 956 (1974).

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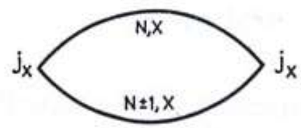
The Kubo formula:

$$\sigma_{xx}=\frac{e^2}{\pi^2\hbar}\,\frac{1}{2}\sum_{N=0}\int\left(-\frac{\partial f}{\partial E}\right)\mathrm{d}E(N+1)\ \mathrm{Im}\,\frac{\hbar\omega_c}{X-(N+1)\hbar\omega_c}\ \mathrm{Im}\,\frac{\hbar\omega_c}{X-N\hbar\omega_c}$$

The only (loop) diagram:



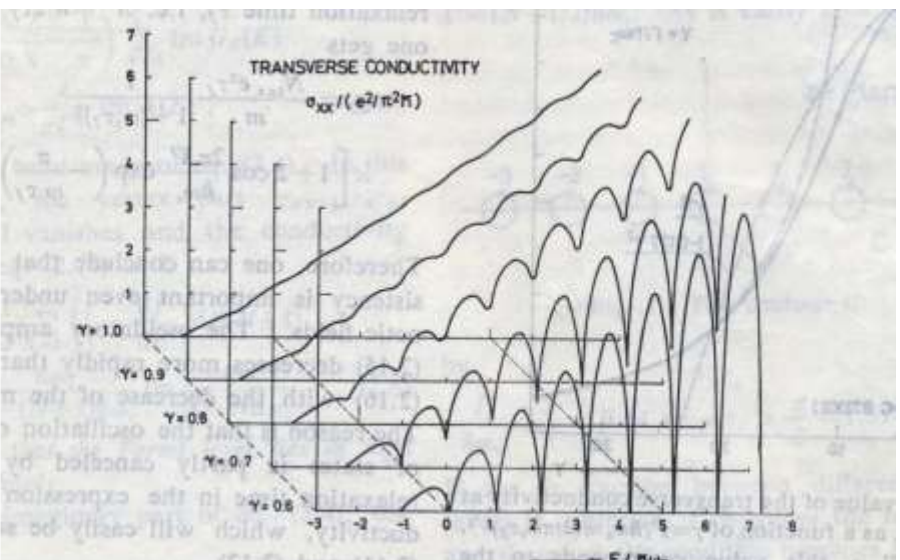


Fig. 4. The oscillation of the transverse conductivity in the SCBA.

Result:

Large fields (separated Landau levels) :

$$\sigma_{xx}(\varepsilon)\approx\frac{e^2}{\pi^2\hbar}N\left[1-\left(\frac{\varepsilon-\varepsilon_N}{\Gamma}\right)^2\right]$$

Intermediate fields :

$$\sigma_{xx}(\varepsilon)=\frac{n_{\mathrm{e}}e^2\tau_q}{m}\frac{z(\varepsilon)^2}{z(\varepsilon)^2+(\omega_{\mathrm{c}}\tau_q)^2}$$

Quantum lifetime and the transport time coincide.

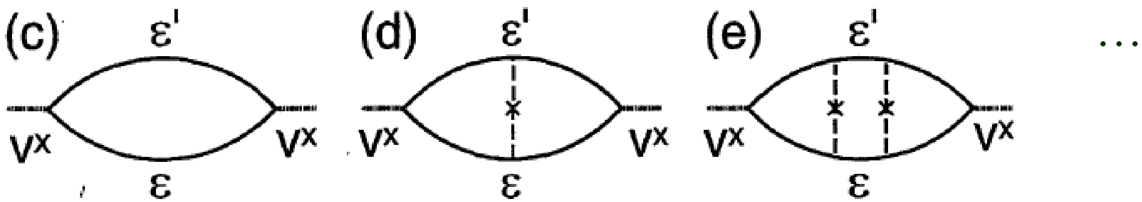
Small (classical) fields : $z=1$.

Quantum conductivity of 2D electron gas in graphene

N. H. Shon and T. Ando, Journ. of the Phys. Soc. of Japan 67, 2421 (1998).

$$\sigma_{xx}(\varepsilon) = \frac{e^2\hbar}{\pi L^2}\mathrm{Tr}\left\langle v^x \mathrm{Im}G(\varepsilon + \mathrm{i}0) v^x \mathrm{Im}G(\varepsilon + \mathrm{i}0)\right\rangle$$

In zero magnetic field the anisotropy of scattering, which is character for graphene, leads to arising of the series of the ladder diagrams instead of the only loop diagram.



$$(s\boldsymbol{k}|U|s\boldsymbol{k}') = \frac{1}{2}u_0(1 + \exp[\mathrm{i}\varphi(\boldsymbol{k} - \boldsymbol{k}')])$$

The quantum lifetime and the transport time do not coincide:

$$\tau_{\mathrm{q}}(\varepsilon) = \left(\frac{\kappa\varepsilon}{2\hbar^3 s^2}\right)^{-1}$$

$$\tau_{\mathrm{tr}}(\varepsilon) = \left(\frac{\kappa\varepsilon}{4\hbar^3 s^2}\right)^{-1}$$

In non-zero magnetic field magnetoconductivity is calculated by the series of the similar ladder diagrams.

Quantum conductivity of 2D electron gas in graphene

The result of summing up the ladder diagrams in magnetic field .

$$\sigma_{xx}(\varepsilon) = N_{\mathrm{deg}} \frac{e^2 n(\varepsilon) 2\tau_q(\varepsilon)}{m(\varepsilon)} \frac{z(\varepsilon)^2}{z(\varepsilon)^2 + [2\omega_{\mathrm{c}}(\varepsilon)\tau_q(\varepsilon)]^2}$$

z depends on ε by the two ways: through the Shubnikov oscillations and due to the slow dependences of all the electron parameters on energy: $z(\varepsilon) = z[\omega_{\mathrm{c}}(\varepsilon)\tau_q(\varepsilon),\ \varepsilon/\hbar\omega_{\mathrm{c}}]$

Let us average the conductivity over the fast oscillation with the periods: $\hbar\omega_{\mathrm{c}}(\varepsilon)$.

Interval of averaging: $\varepsilon_{\mathrm{F}} \gg \Delta\varepsilon(\varepsilon) \gg \hbar\omega_{\mathrm{c}}(\varepsilon)$

We obtain the averaged conductivity $\sigma_{xx}^{\mathrm{av}}(\varepsilon)$, depending on energy only through the parameter $x = \omega_c(\varepsilon)\tau_q(\varepsilon)$.

$T \gg \hbar\omega_{\mathrm{c}}(\varepsilon)$

$$\sigma_{xx}^{\mathrm{av}}(\varepsilon) = \sigma_{xx}^{\mathrm{D}}[\omega_{\mathrm{c}}(\varepsilon)\tau_q(\varepsilon)]\ \eta_{xx}[\omega_{\mathrm{c}}(\varepsilon)\tau_q(\varepsilon)]$$

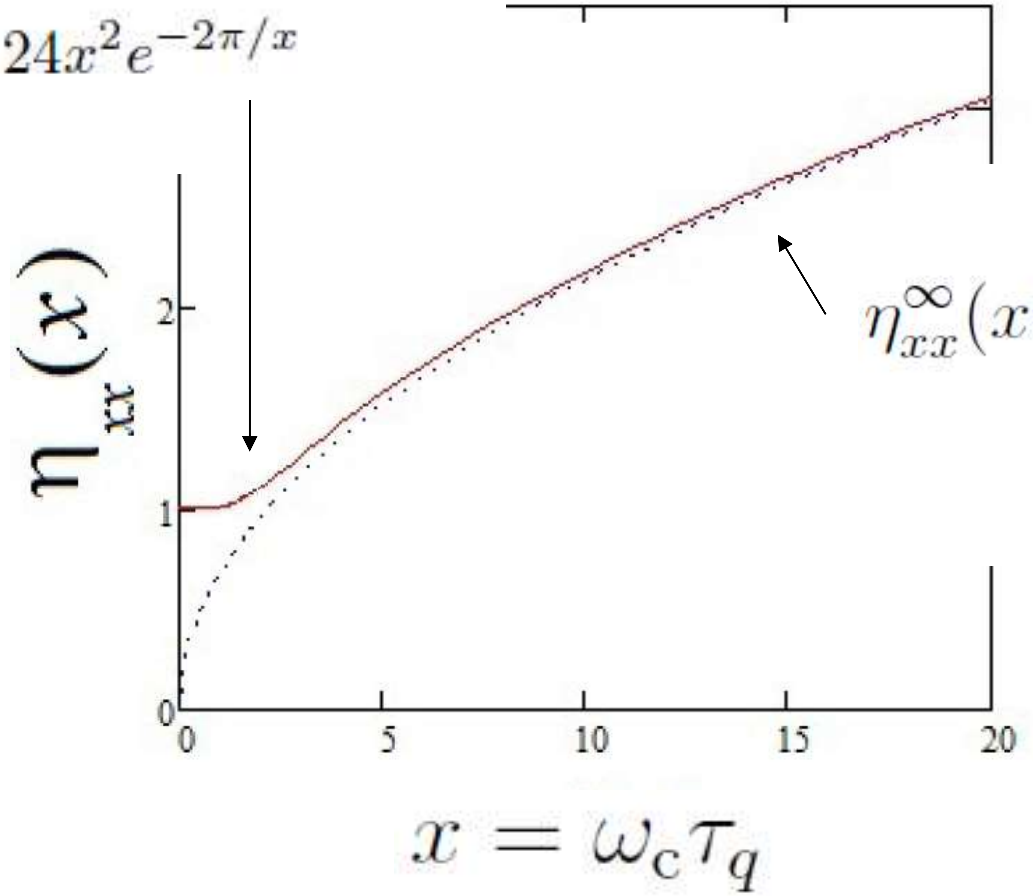
Classical Drude conductivity, which, in fact, depends on $\omega_c\tau_{tr} = 2\omega_c\tau_q$

Dimensionless function describing quantum-mechanical nature of magnetoconductivity of 2D electron gas

Quantum conductivity of 2D electron gas in graphene

$$\eta_{xx}(x) = 1 - 24x^2 e^{-2\pi/x}$$

for $x \ll 1$:



$$\eta_{xx}^{\infty}(x) = \frac{8\sqrt{2}}{3\pi\sqrt{\pi}}\sqrt{x}$$

for $x \gg 1$

A trivial conclusion: for high H when $\omega_c\tau_q \,|_{\max(E\_F \text{ or } T)} \gg 1$

we have : $\rho/\rho_0 \sim (\rho/\rho_0)_{Drude}\,(\omega_c\tau_q)^{1/2}$

Comparing of quantum and classical magnetoconductivity of grahene

So the total conductivity takes again the form:

$$\sigma_{xx} = \sigma_{xx}^{\mathrm{D},0} + \Delta\sigma_{xx}$$

where

$$\sigma_{xx}^{\mathrm{D},0} = A\int_0^\infty d\varepsilon \left(-\frac{\partial n_0}{\partial \varepsilon}\right)(\varepsilon) = A n_0(0) \qquad \Delta\sigma_{xx} = A\varepsilon^* \left(-\frac{\partial n_0}{\partial \varepsilon}(0)\right) I$$

$$\frac{\Delta\sigma_{xx}}{\sigma_{xx}^{\mathrm{D},0}} \sim \sqrt{H}$$

But now: $I = \int_0^\infty d\tilde{\varepsilon}\, \{\tilde{\sigma}_{xx}^{\mathrm{D}}[x(\tilde{\varepsilon})]\eta_{xx}[x(\tilde{\varepsilon})] - 1\}$, I = -1.568

A rather surprising fact I_{quantum} and $I_{\text{cclassical}}$ differ very little.

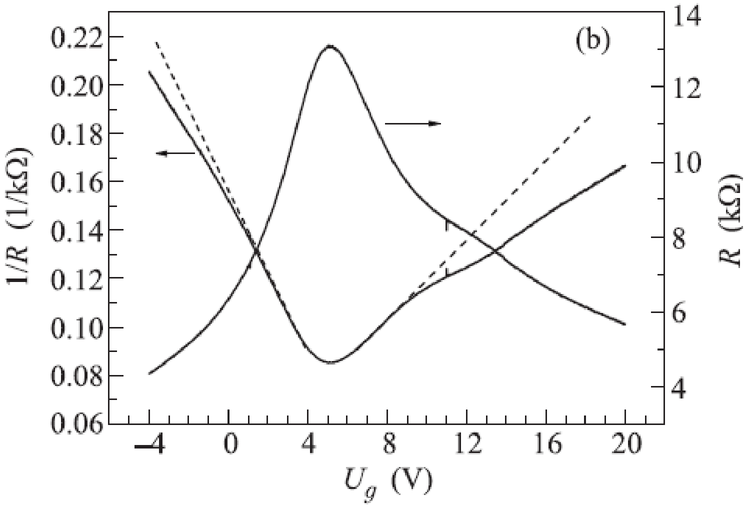
An analysis shows that the contribution of $\sigma_{xy}(H)$ is not substantial in magnetoresistance, so:

$$\frac{\Delta\varrho_{xx}}{\varrho_{xx}^0} = -2\sqrt{\frac{e\gamma}{c}s}\left(-\frac{\partial n_0}{\partial \varepsilon}(0)\right) I\sqrt{H}$$

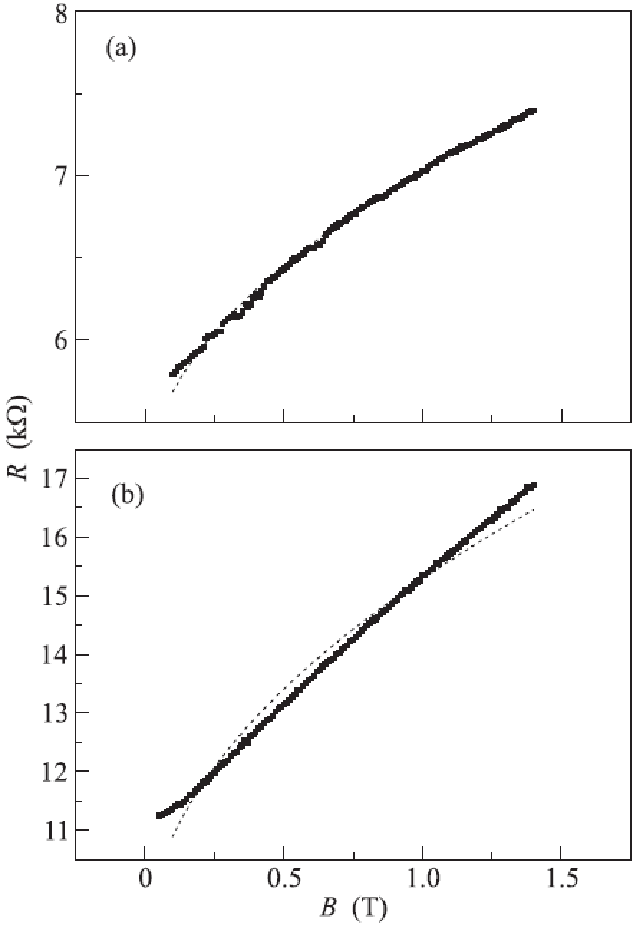
"2" comes from taking into account the hole states.

P.S. Alekseev, A.P. Dmitriev, I.V. Gornyi, V.Yu. Kachorovski, Phys. Rev. B 2013.

Experiment



This dependence $\sigma(U_g, H=0)$ shows that scatteringon impurities with the short-range potential does not dominate but does present.



Magnetoresistance scales as $\sim H^{1/2}$ for U_g corresponding ε_F far from “ the neutrality point”

and

do not scales as $\sim H^{1/2}$ for U_g corresponding mu near “the neutrality point”.

G.Yu. Vasil’eva, P.S. Alekseev, Yu.L. Ivanov, Yu.B. Vasil’ev, D. Smirnov, H. Schmidt, R.J. Haug, F. Gouider, G. Nachtwei, JETPh Lett. 2012.

Conclusion

Resistance of disordered graphene (impurities with the short-range potential) at high temperatures and enough low magnetic fields consists on the two contribution:

(i) the main Drude contribution, independent on a magnetic field, and

(ii) a small correction, which depends on magnetic field as a square root and comes from the carriers with energies near the degenerated point (ε_k=0) of the graphene spectrum.