Orbital Lamb shift in the lowest Landau levels of gaphene bilayers and trilayers

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1. Introduction

Graphene $\supset$ **Dirac fermions** $\rightarrow$ unique electronic properties

Increased interest in **bilayer and few-layer graphene**

Layers $\rightarrow$ Physics and applications of graphene **richer**

bilayer graphene : tunable band gap

Notable features of bilayer graphene:

- Presence of **zero-energy Landau levels** $\leftarrow$ **Dirac fermions** in B

  Graphene $\supset$ 4 zero-energy LLs: spin & valley

  Bilayer graphene $\supset$ **8 zero-energy LLs**

  also **degen.** in **orbital** ***n*** **= (0,1)**

- **Orbital degeneracy** $\leftarrow$ **layers + topology**

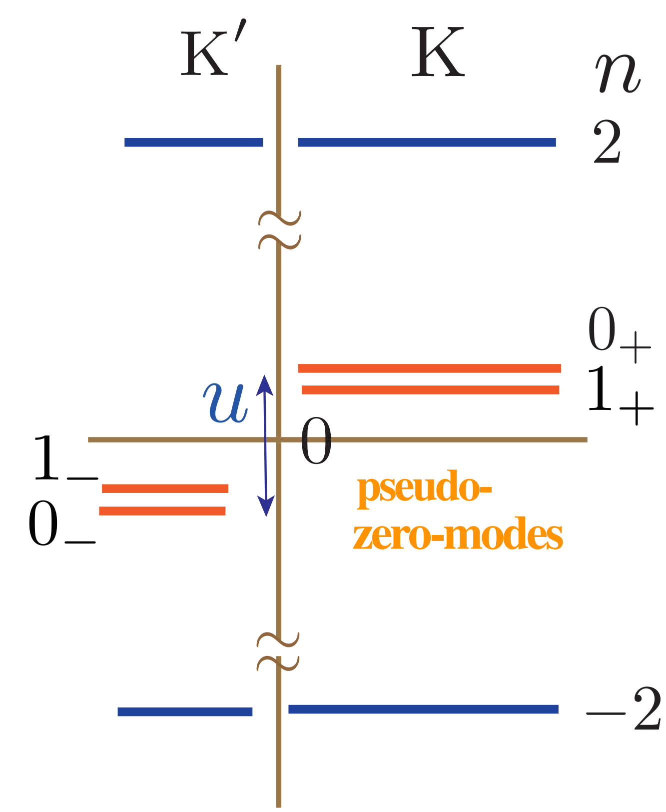
  (nonzero **index** of *H*)

  In real samples

  Zero-energy LLs $\rightarrow$ **Pseudo-zero-mode (PZM) LLs**

  $\rightarrow$ Fine structure of the **Lowest LLs**

  {PZM levels} = LLLs



LLLs in bilayer graphene

- Graphene: "**many-body**" system

  strong quantum fluctuations of the **valence band** $\rightarrow$ **Renormalization, Corrections to CR**, ...

Here we report [1] **a new kind of**
many-body quantum effects in bilayer (& few-layer) graphene :

(1) Orbital degeneracy of the PZM levels is **lifted**
by **Coulombic quantum fluctuations of the valence band** (the Dirac sea).
→ analogous to the **Lamb shift in** the **hydrogen** atom
($2P_{1/2}$ - $2S_{1/2}$ splitting ← vacuum fluctuations)

(2) This **"orbital" Lamb shift** is a **vacuum effect** but
is correlated with the Coulomb interaction within the LLLs

→ **Orbital mixing : "Lamb-shifted" orbital modes,** with filling, **get mixed.**

(1) + **(2)** essentially **govern the structure** & **spectrum of** the **LLLs** of bilayers.

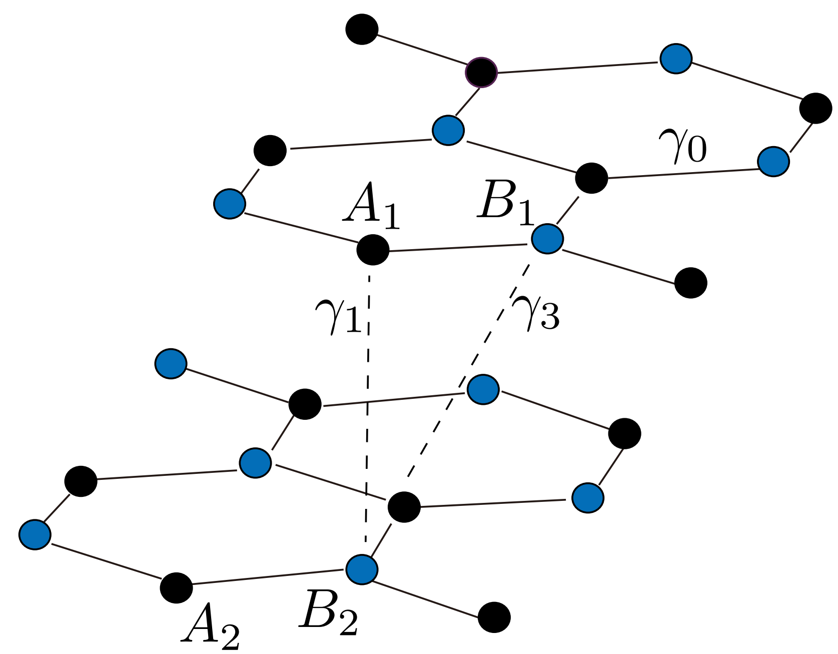
(3) Experimental signatures : Interaction-enhanced **spin/valley or orbital gaps**
observed via **QHE** within the LLLs

[1] K. Shizuya, PRB 86, 045431 (2012)

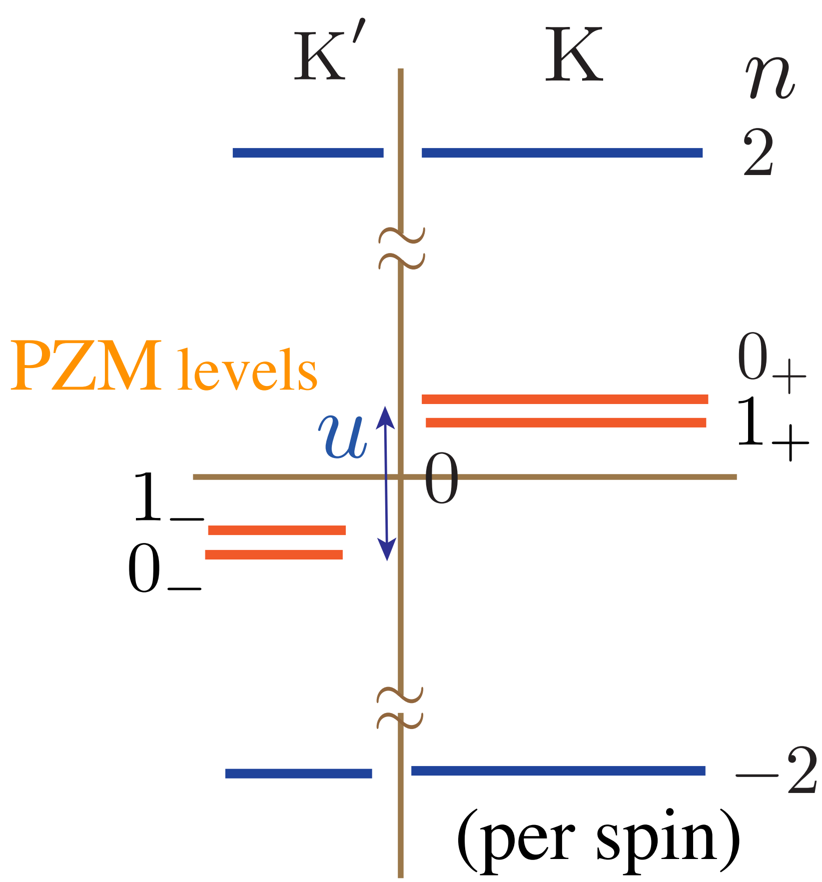
2. bilayer graphene

$$H^{\rm bi} = \int d^2\mathbf{x}\, \left[\Psi^\dagger \mathcal{H}_K \Psi + \tilde{\Psi}^\dagger \mathcal{H}_{K'}\tilde{\Psi}\right] + V^{\rm Coul}$$

$$\mathcal{H}_K = \begin{pmatrix} \frac{1}{2}u & & & v\,p^\dagger \\ & -\frac{1}{2}u & v\,p & \\ & v\,p^\dagger & -\frac{1}{2}u & \gamma_1 \\ v\,p & & \gamma_1 & \frac{1}{2}u \end{pmatrix}, \quad \Psi = \begin{pmatrix} \psi_{A_2} \\ \psi_{B_1} \\ \psi_{A_1} \\ \psi_{B_2} \end{pmatrix}$$



$v \approx 1.1 \times 10^6$ m/s $\qquad \gamma_1 \equiv \gamma_{A_1 B_2} \sim 0.4\,\mathrm{eV}$



(i) **Pseudo zero-mode LLs** (K/K')

u :interlayer bias $\longrightarrow$ valley breaking

$n = 0_\pm \qquad \epsilon_0^u = \pm\frac{1}{2}u$

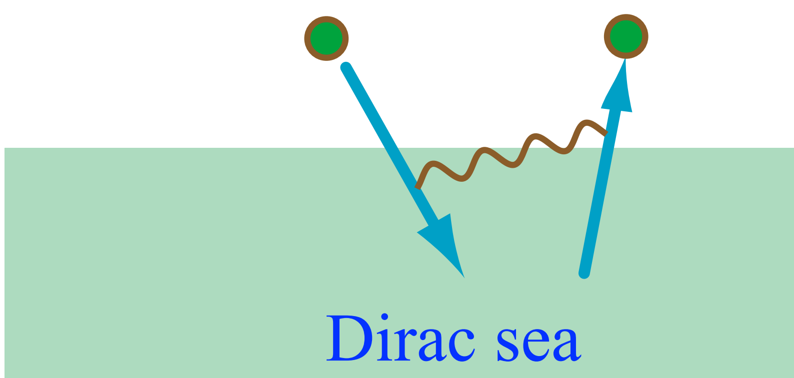
$n = 1_\pm \qquad \epsilon_1^u = \pm\frac{1}{2}u(1-z)$ $(z \sim 0.2$ for $B = 10\mathrm{T})$

(ii) Coulomb interaction

$$V^{\mathrm{Coul}} = \frac{1}{2}\sum_{\mathbf{p}} v_{\mathbf{p}} : \rho_{-\mathbf{p}}\rho_{\mathbf{p}} : , \qquad \rho_{-\mathbf{p}} = \gamma_{\mathbf{p}} \sum_{k,n=-\infty}^{\infty} g_{\mathbf{p}}^{kn} \int dy_0\, \psi_k^\dagger(y_0)\, e^{i\mathbf{p}\cdot\mathbf{r}} \psi_n(y_0)$$

$$\gamma_{\mathbf{p}} = e^{-\frac{1}{4}\ell^2\mathbf{p}^2}$$

- Calculate **quantum fluctuations of the valence band**
  Construct a **PZM eff. theory** over the **filled valence band - - - HF approx.**



$|\mathrm{DS}\rangle$ $n \le -2$ occupied

$$\langle \mathrm{DS}|V|\mathrm{DS}\rangle \sim \epsilon_0^{\mathrm{v}}\, (\psi^\dagger\psi)_{n=0} + \epsilon_1^{\mathrm{v}}\, (\psi^\dagger\psi)_{n=1}$$

$$\epsilon_0^{\mathrm{v}} > \epsilon_1^{\mathrm{v}}$$

$$\epsilon_0^{\mathrm{v}} = \sum_{\mathbf{p}} v_{\mathbf{p}}\gamma_{\mathbf{p}}^2\, \tfrac{1}{2}(|g_{\mathbf{p}}^{00}|^2 + |g_{\mathbf{p}}^{01}|^2) \sim 0.73\, \tilde{V}_c$$

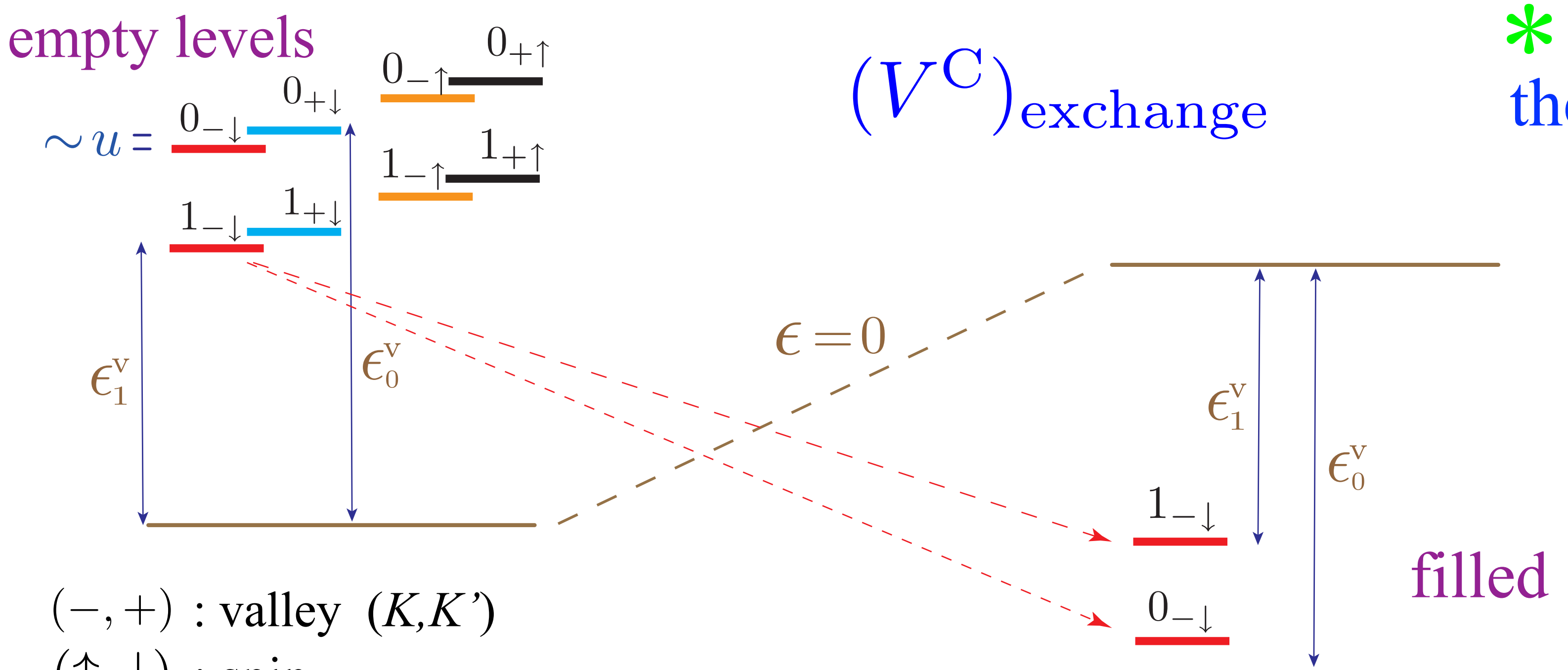
$$\epsilon_1^{\mathrm{v}} = \sum_{\mathbf{p}} v_{\mathbf{p}}\gamma_{\mathbf{p}}^2\, \tfrac{1}{2}(|g_{\mathbf{p}}^{11}|^2 + |g_{\mathbf{p}}^{10}|^2) \sim 0.58\, \tilde{V}_c$$

$$\tilde{V}_c = \sqrt{\frac{\pi}{2}}\,\frac{e^2}{\ell}$$

Orbital degeneracy lifted by **quantum fluctuations of the Dirac sea**
$\longrightarrow$ Quantum splitting of the PZM levels in BLGR, analogous the **Lamb shift**(*)
"Orbital Lamb shift"

c.f. **Lamb shift**(*) : fine structure of hydrogen **Lamb-Retherford (1947)**
Energy difference between the $2\mathrm{P}_{1/2}$ and $2\mathrm{S}_{1/2}$ states due to vacuum fluctuations.
1057.8MHz Dirac theory ($E_{n,j}$, $\mathbf{j} = \mathbf{l} + \mathbf{s}$) $\rightarrow$ **QED**

(iii) **Coulomb exchange int.** within **the PZM sector** (~ diag. in spin & valley)



$(-,+)$: valley (K,K')

$(\uparrow,\downarrow)$: spin

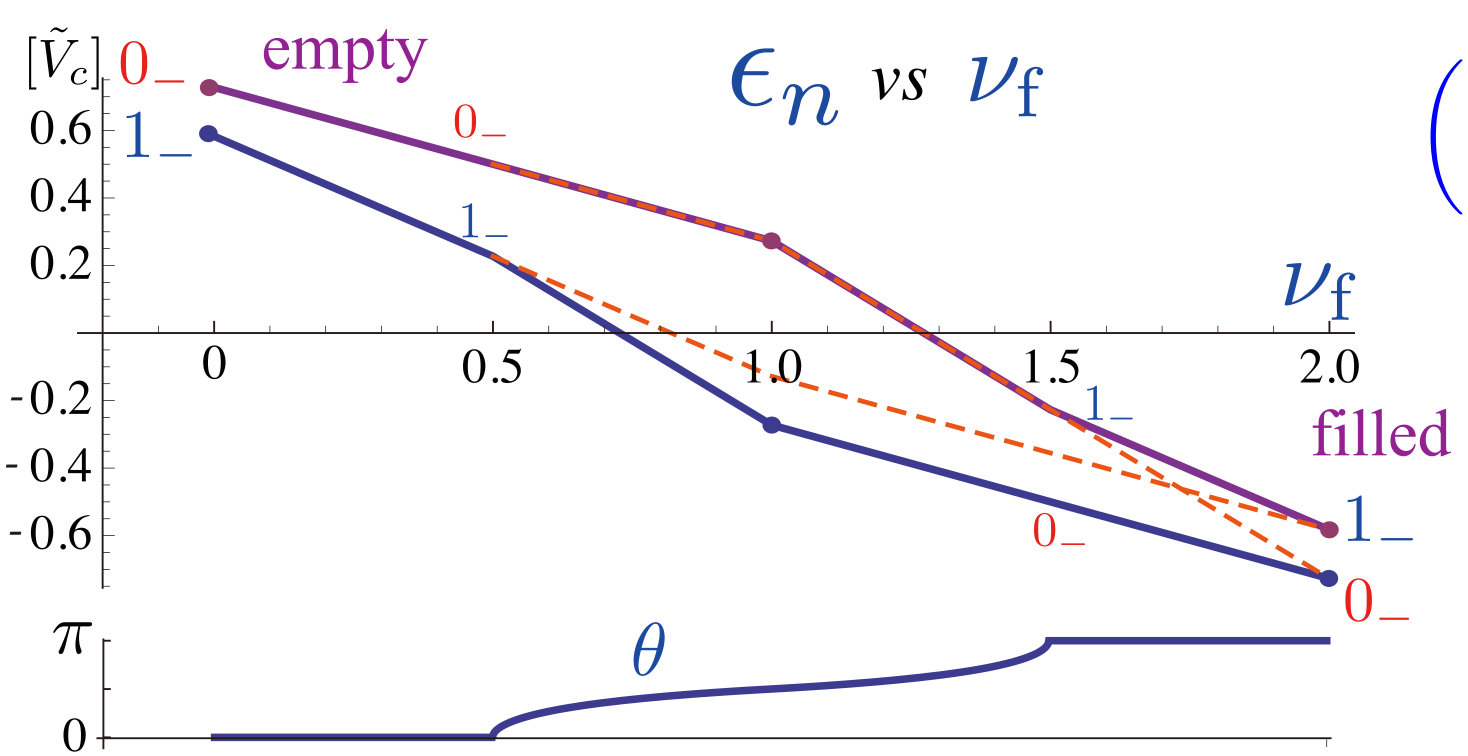
* When the PZM sector is **filled** up, the energy of each level changes **sign**:

* both $n = 0,1$ filled

$$\epsilon_{0-}^{\rm v} \;\to\; -|\epsilon_{0-}^{\rm v}|$$

$$\epsilon_{1-}^{\rm v} \;\to\; -|\epsilon_{1-}^{\rm v}|$$

- **How does each level evolve**? Diagonalize $H^{\rm HF}$ by a rotation in orbital $n = (0,1)$ space



$$\begin{pmatrix} \Psi_{0-} \\ \Psi_{1-} \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2} & -e^{-i\phi}\sin\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} \Phi_{0-} \\ \Phi_{1-} \end{pmatrix}$$

0_-(empty) $\to 1_-$(filled)

1_-(empty) $\to 0_-$(filled)

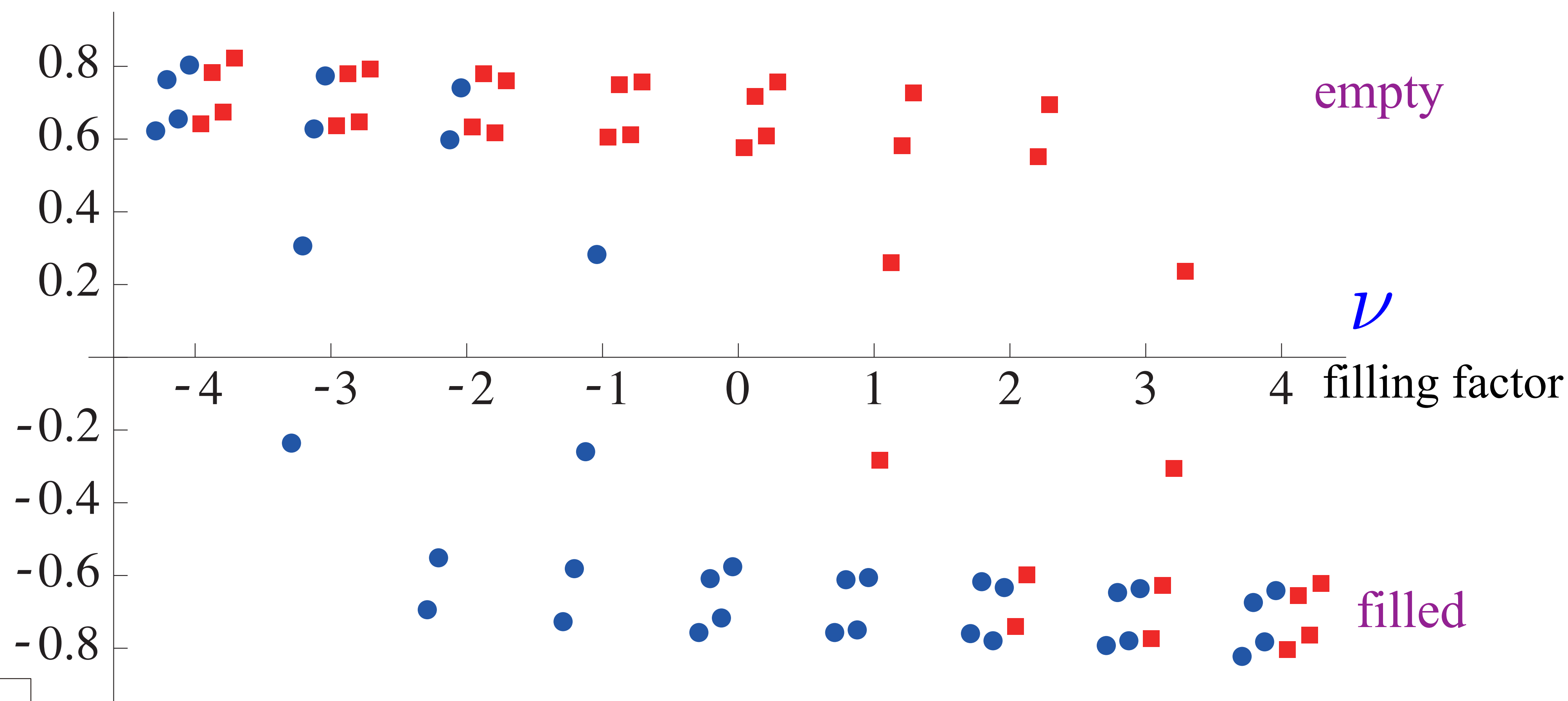
* **Orbital mixing** takes place with filling of the PZM levels

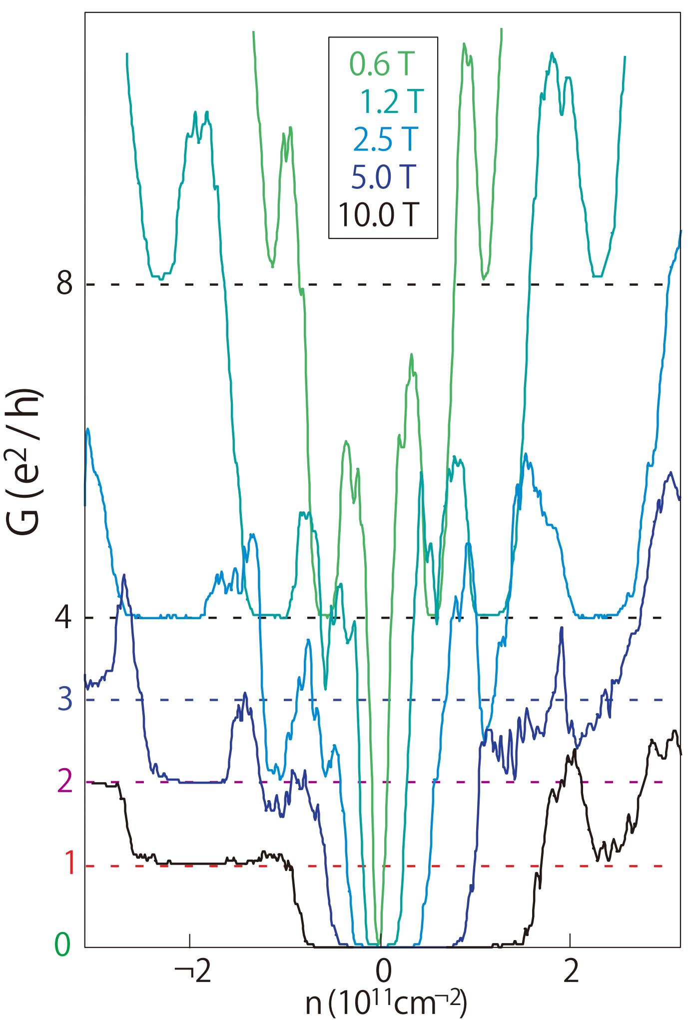
Orbital mixing without Level crossing : Orbital gaps survive

Spectrum of the LLLs vs ν

PZM spectrum

e-h symmetric





Large gaps at $\nu = \pm 2$ & $\nu = 0$: Coulomb-enhanced spin/valley gaps

$\nu = 0$ gap > $\nu = \pm 2$ gap > $\nu = \pm 1, \pm 3$ orbital gap (bias u ~0)

Broken-symmetry states: observable via **QHE**

(& via **Cyclotron Resonance**)

Experimentally: 8-fold degeneracy of the LLLs is lifted

← B.E. Feldman, J. Matin, A.Yacoby, Nat. Phys. ; . . .

3. Conclusion

In the **LLLs of bilayer graphene, there arise**

- **Orbital Lamb shift** **(Orbital degeneracy: lifted by a many-body effect)**

  → **orbital breaking > valley/spin breaking**

- **Orbital mixing without level crossing** : orbital gaps survive.

  Interaction-enhanced spin/valley **gaps** > $\nu = \pm 1, \pm 3$ orbital **gaps**

* **These effects, though unnoticed earlier,**
govern the structure and spectrum of the LLLs in BLGR.

Experimental signatures : **Interaction-enhanced gaps** observed via **QHE**

e.g., $\nu = 0$ insulating state

- Generalization to **trilayers**: similar results for *ABC*-stacked trilayer GR

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