A novel mesoscopic phenomenon: An analog of the Braess paradox in 2DEG networks

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Outline

- Braess paradox in classical networks

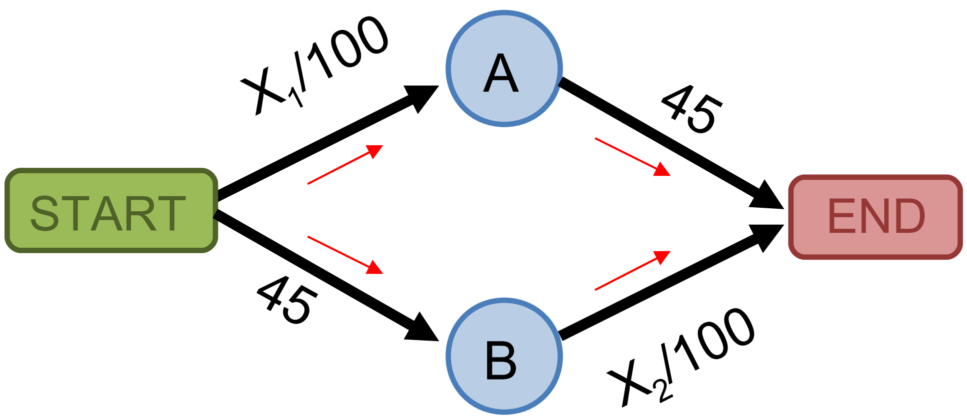
- Simulations of a quantum network

- Simulations of scanning gate experiments

- SGM experiments on a real network

The original Braess paradox for road networks

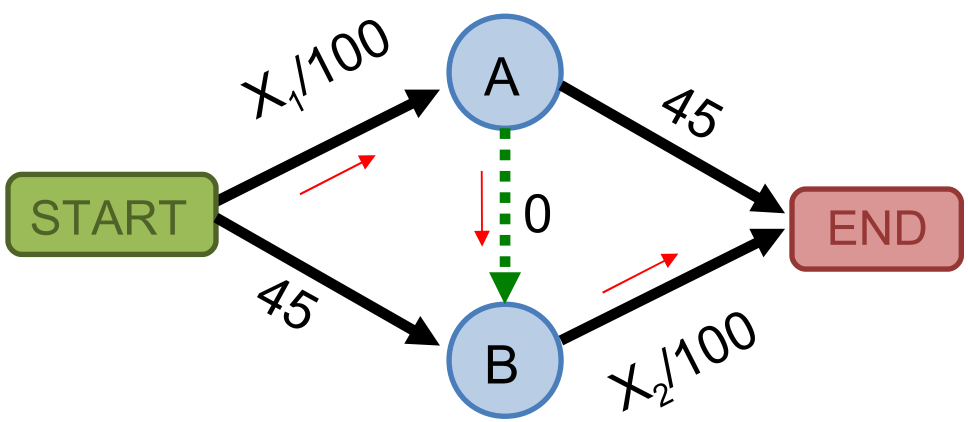
*Braess (1968): Adding extra road to a **congested** network, where the moving entities freely choose their route, can in **some** cases reduce overall performance.*



$X = X_1+X_2 =$ **4000 drivers** want to go from **START** to **END** while freely choosing their way

(Nash) equilibrium : $X_1 = X_2 = 2000$

Total travel time for each driver = 65 min



A high-speed road with **0 min** travel time is opened from A to B

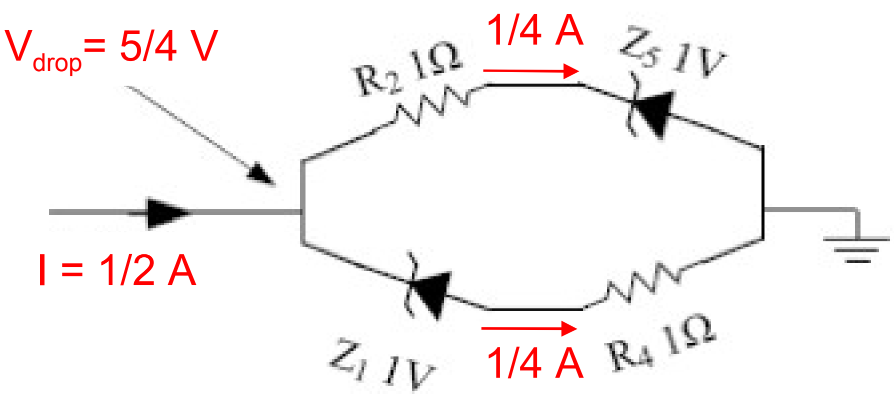
New (Nash) equilibrium : all drivers choose the **Start > A > B > End** road : $X_1 = X_2 = 4000$

Travel time for each driver = 80 min !!!

This paradox is explained by ***game theory*** :
Each traveler chooses an ***individual*** optimum that is not the ***social*** optimum.

Braess paradox in electrical networks

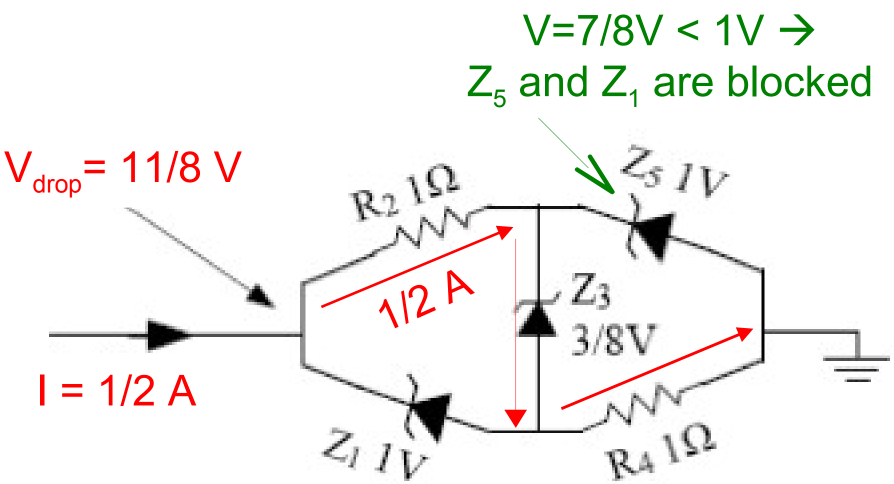
See: Cohen & Horowitz, *Nature* (1991) and Penchina & Penchina, *Am. J. Phys.* (2003)



A Wheastone bridge with **non-linear** Zener diodes at constant current = 1/2 A

At equilibrium, a 1/4 A current flows in each branch: voltage drop = 5/4 V

Z = 2.5 Ω



An additional (lower-voltage) Zener bypasses the two branches.

At equilibrium, the entire current flows along red arrows (1V Zener diodes are blocked): voltage drop = 11/8 V > 5/4 V

Z = 2.75 Ω

Braess paradox : offering a new path to current increases impedance.

Note: If only linear components are used, no paradox occurs.

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Braess paradox in the quantum world ???

Known so far for classical networks only (road, electric, hydraulic, thermal)

we propose to **extend the Braess paradox to the quantum world**

using **quantum simulations** and **Scanning Gate Microscopy**

M.G. Pala *et al.*, *Physical Review Letters* **108**, 076802 (2012)

M.G. Pala *et al.*, *Nanoscale Research Letters* **7**, 472 (2012)

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Simulations of a rectangular mesoscopic network

Rectangular corral :
$L_x \times L_y = 1.0 \times 1.6\ \mu m^2$

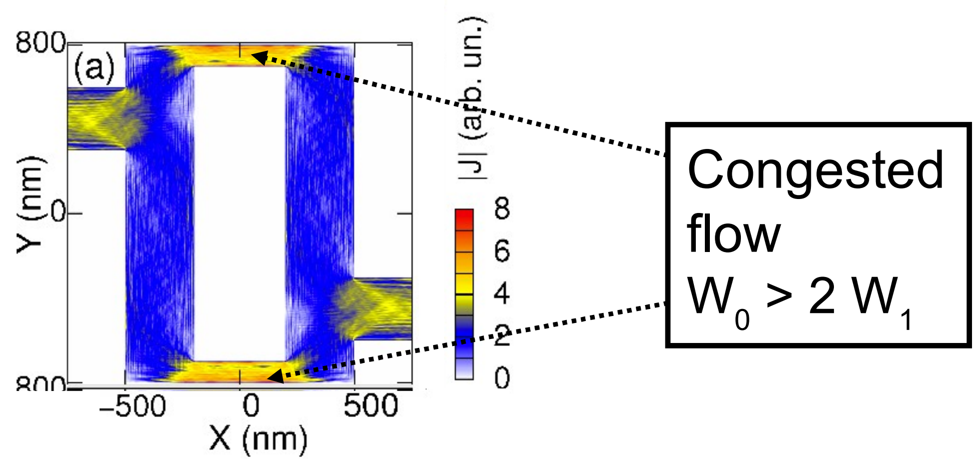
openings: $W_0 = 300$ nm
vertical wires: $W_0 = 300$ nm
horizontal wires: $W_1 = 100$ nm
Fermi wavelength: 20 nm

Ballistic and coherent regime :
L ≤ mean-free path & coherence length

Quantum simulations of the current density distribution :

$$|\mathbf{J}|(X, Y)$$

at T = 4.2 K and $V_{bias} = 1$ mV
Keldysh-Green's function formalism
Marco Pala @ IMEP (Grenoble)



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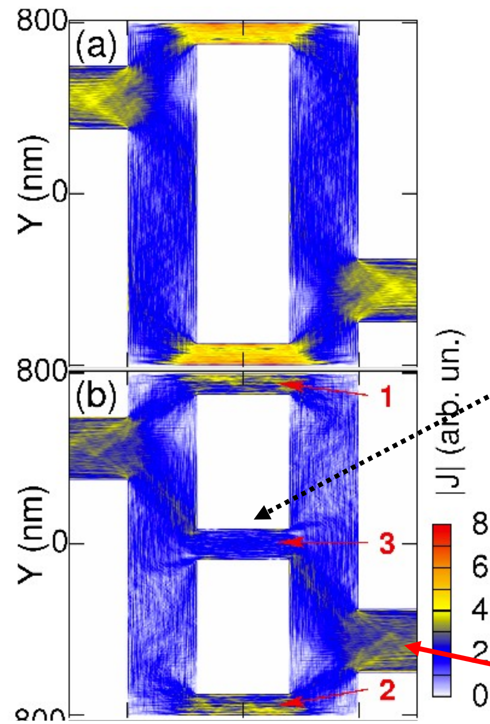

Additional wire 150 nm

Transmitted current decreases !

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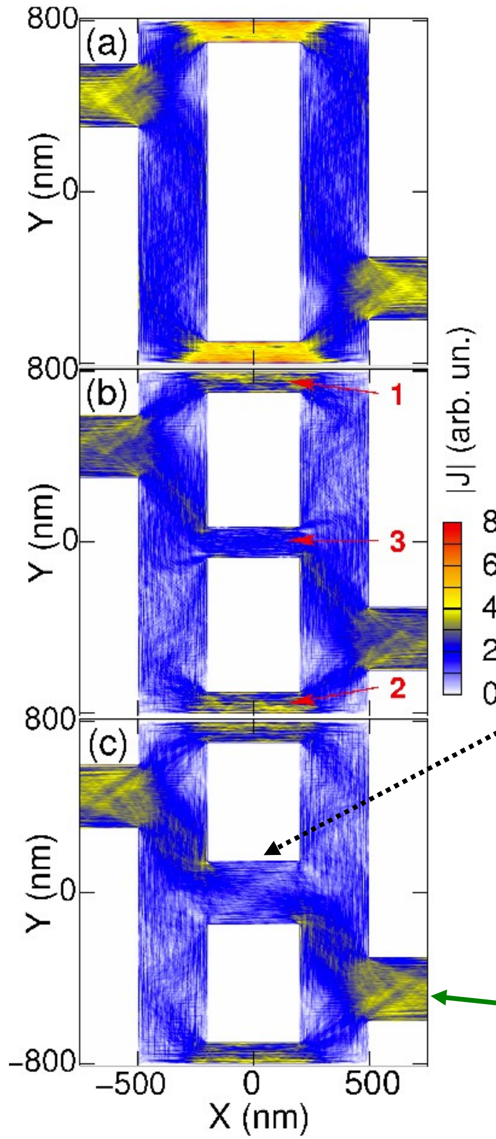

Additional wire 300 nm

Current now increases

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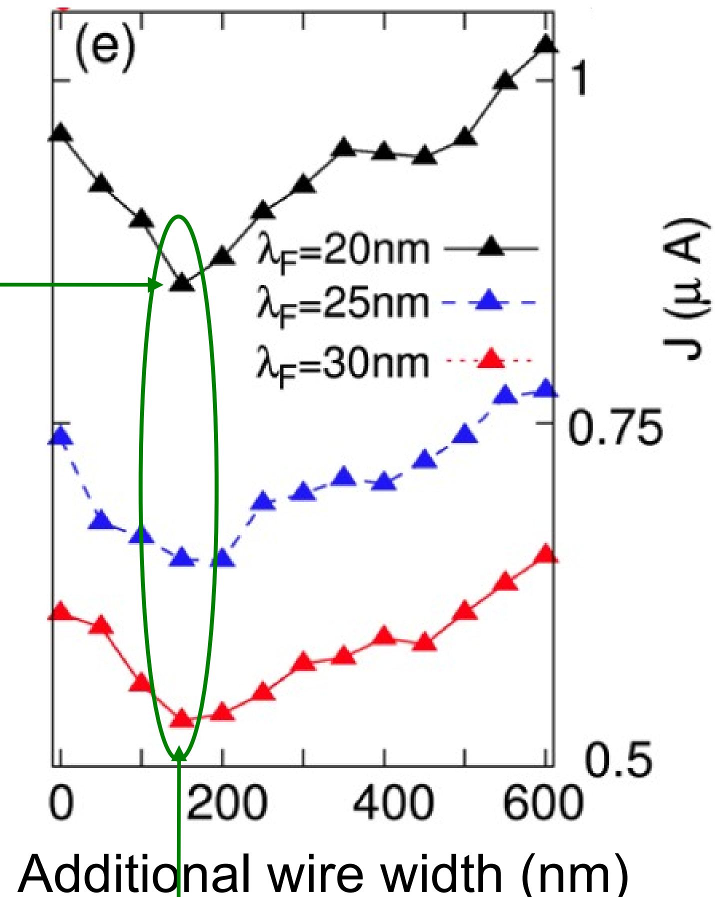
Simulations of a rectangular mesoscopic network

Total transmitted current for different Fermi wavelengths

The total current drops when additional channels are added to the network :

→ a Braess like paradox !



The current minimum is at the same width irrespective of λ_F :

→ NOT a quantum interference effect !

How to measure the effect ?

Patterning a series of devices with different widths would give anoying mesoscopic fluctuations...

It would be better to use a single device with a flying nanoscale gate :
→ **Scanning Gate Microscopy** !!!

Introduction to SGM

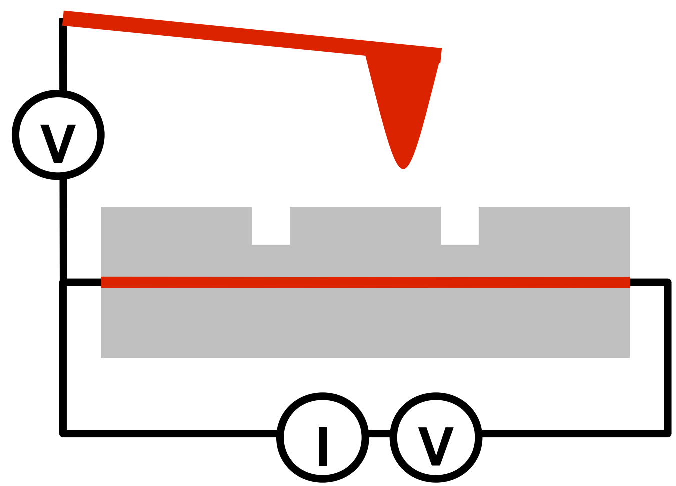
Local probe of electron properties in semiconductor heterostructures

where electrons are several tens of nanometers below the surface
thus not accessible by Scanning Tunneling Microscopy

SGM requires Atomic Force Microscopy techniques for tip positioning

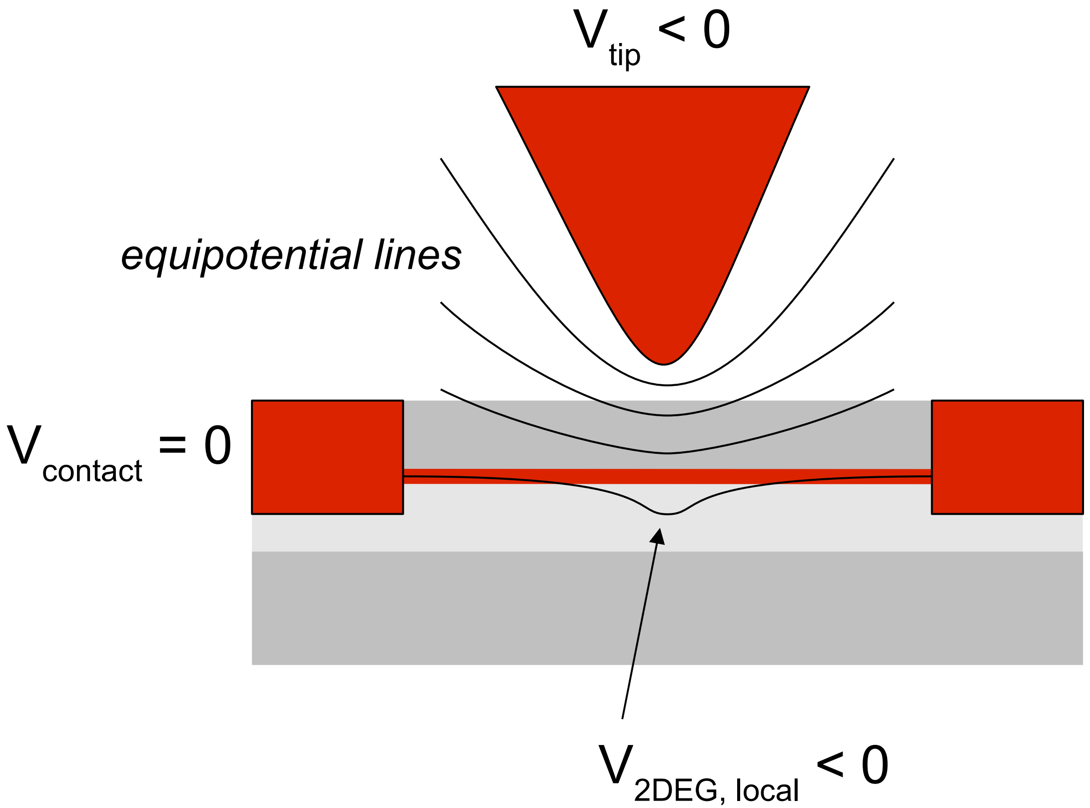
2DEG →

- Quantum Point Contact
- Quantum Wire
- Quantum Dot
- Quantum Ring
- Quantum Hall Effect



Introduction to SGM

Low density electron gas ⇒ imperfect screening of the tip potential
⇒ local potential change ⇒ modified electron scattering ⇒ conductance change



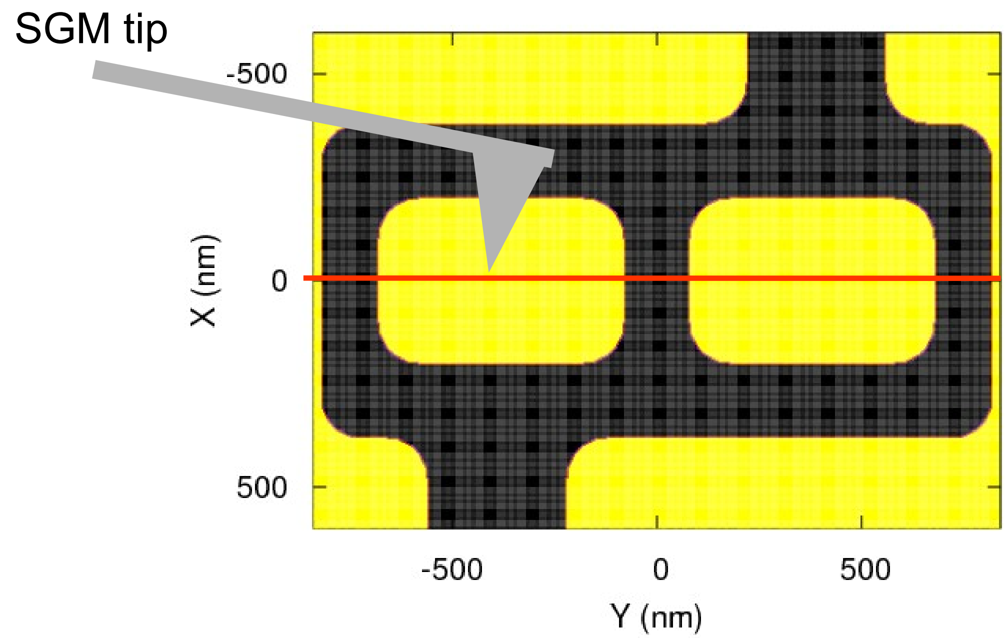
Other ingredients :
Contact potential
Dielectric constants
Etched trenches
Surface gates
Charged defects

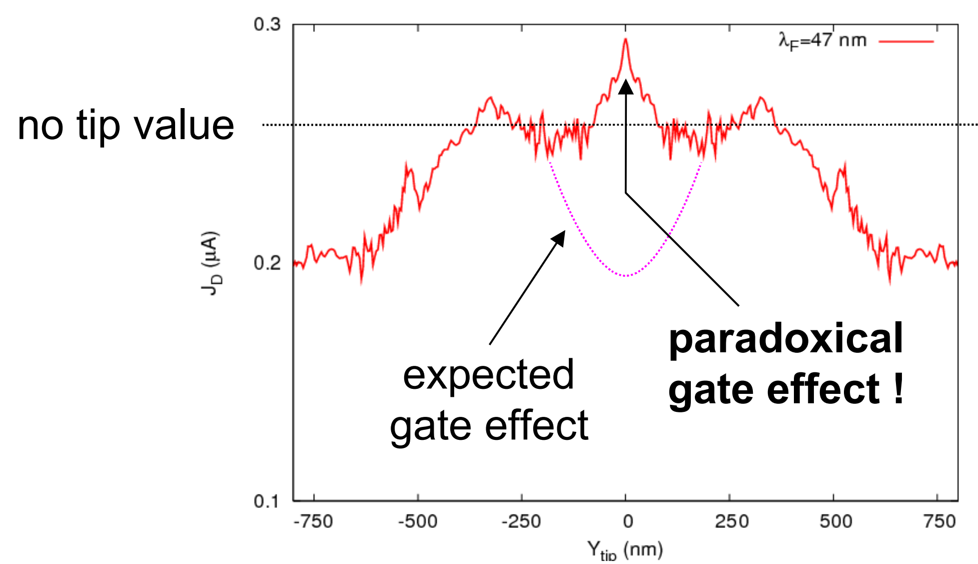
Simulation of SGM experiments

Sample parameters :
openings : W_0 = 300 nm
horizontal wires : W_0' = 180nm
side arms : $W_1 = W_2$ = 140 nm
central arm : W_3 = 160 nm
Fermi wavelength = 47 nm
device bias = 1 mV

Tip parameters :
V_{tip} = -1 V
d_{tip} = 100 nm
Full 2DEG depletion under the tip
(about 300 nm diameter)

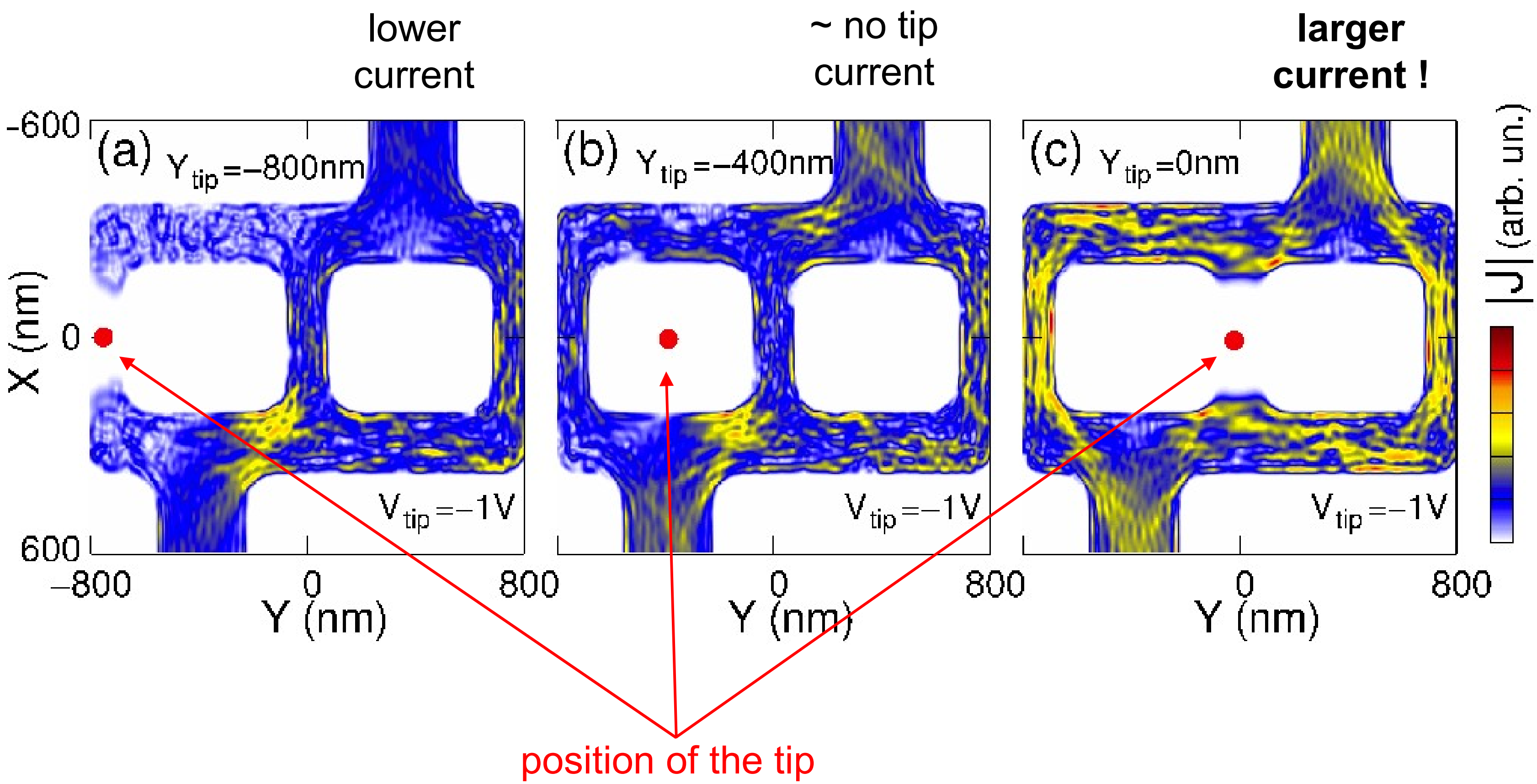
Result :
Current peak above the no tip value
Only at the central arm
Large effect (larger than UCF)





Simulation of SGM experiments

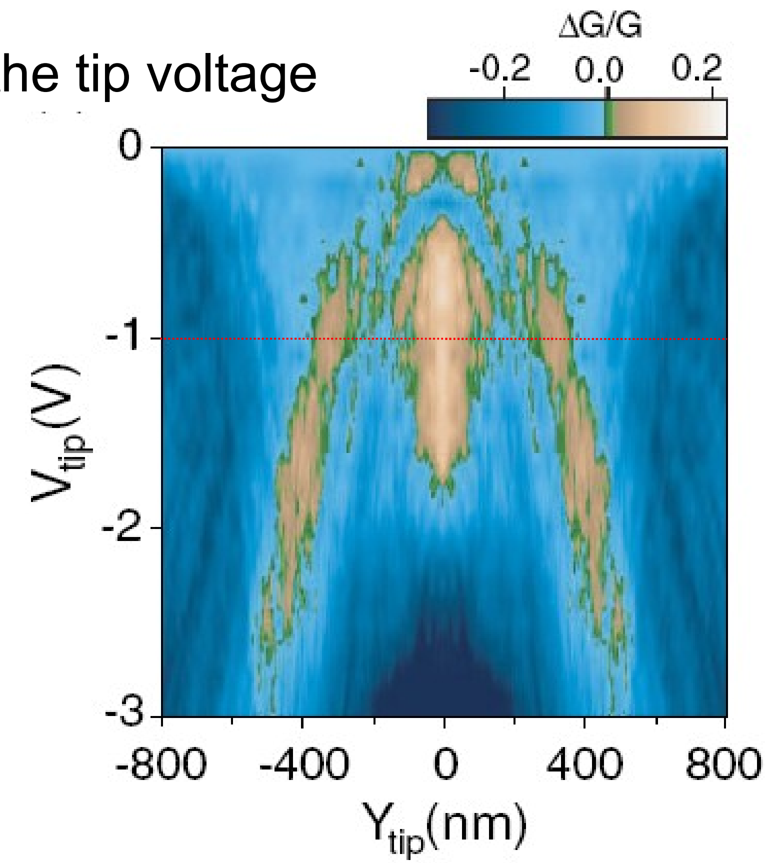
Current density distribution : $|\mathbf{J}|(X, Y)$



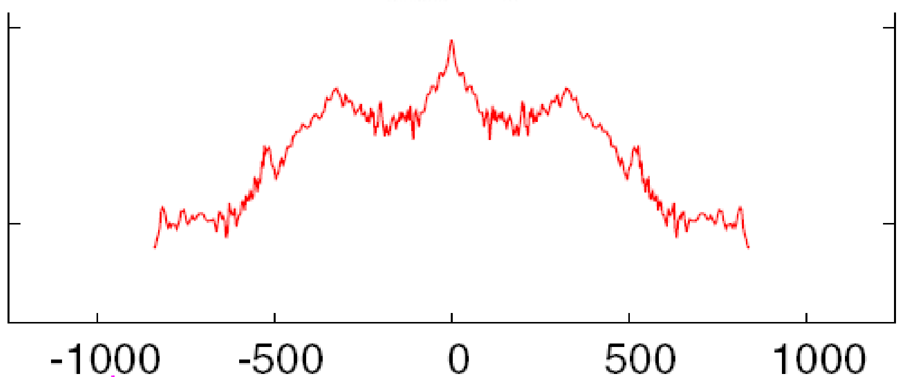
Current flow is enhanced by the **suppression of closed loops** inside the network arms : a single path from injection to exit is preferable !

Simulation of SGM experiments

Influence of the tip voltage

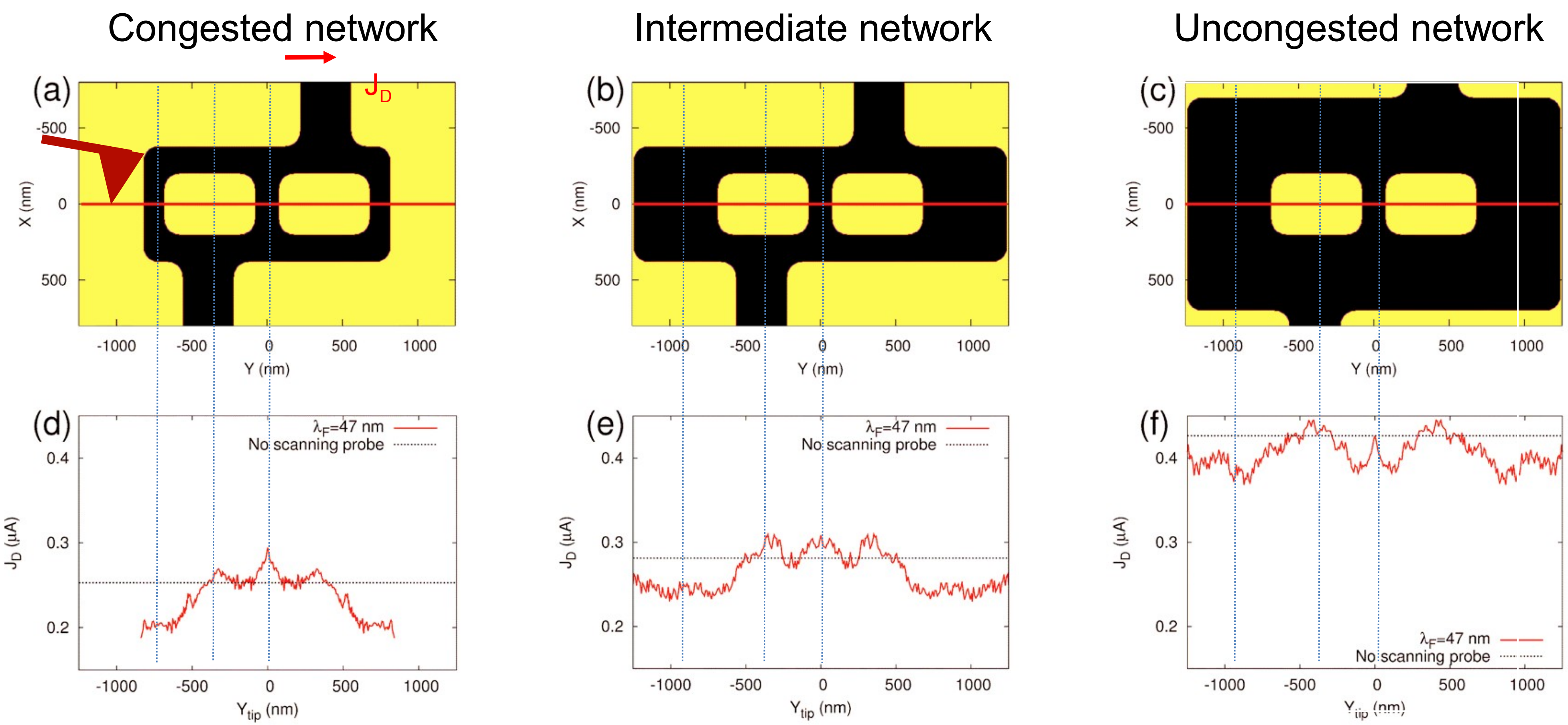


The current peak is only observed in the range V_{tip} = [-0.5; -1.5] V



Simulation of SGM experiments

Role of congestion in the side arms

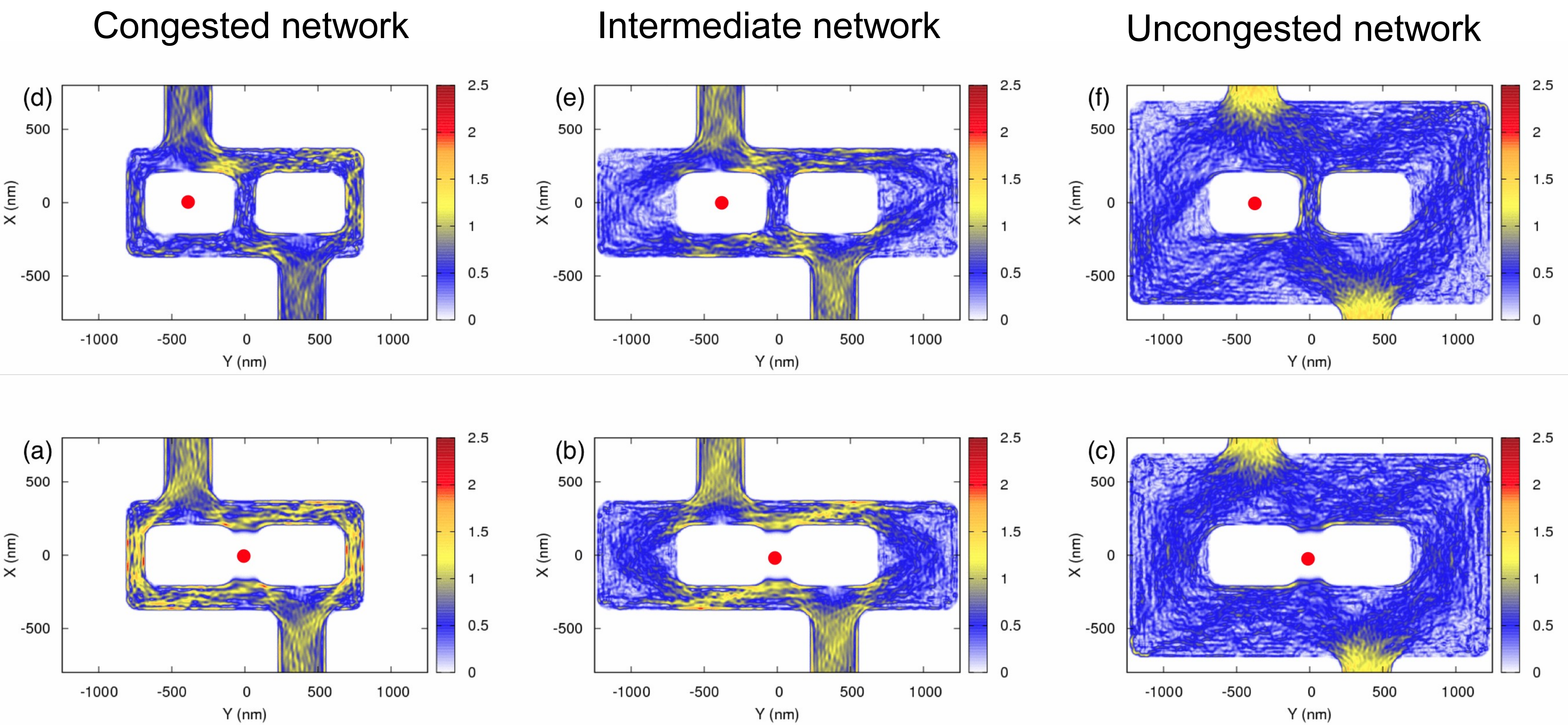


Clear "paradox"

Intermediate case

Residual paradox ?

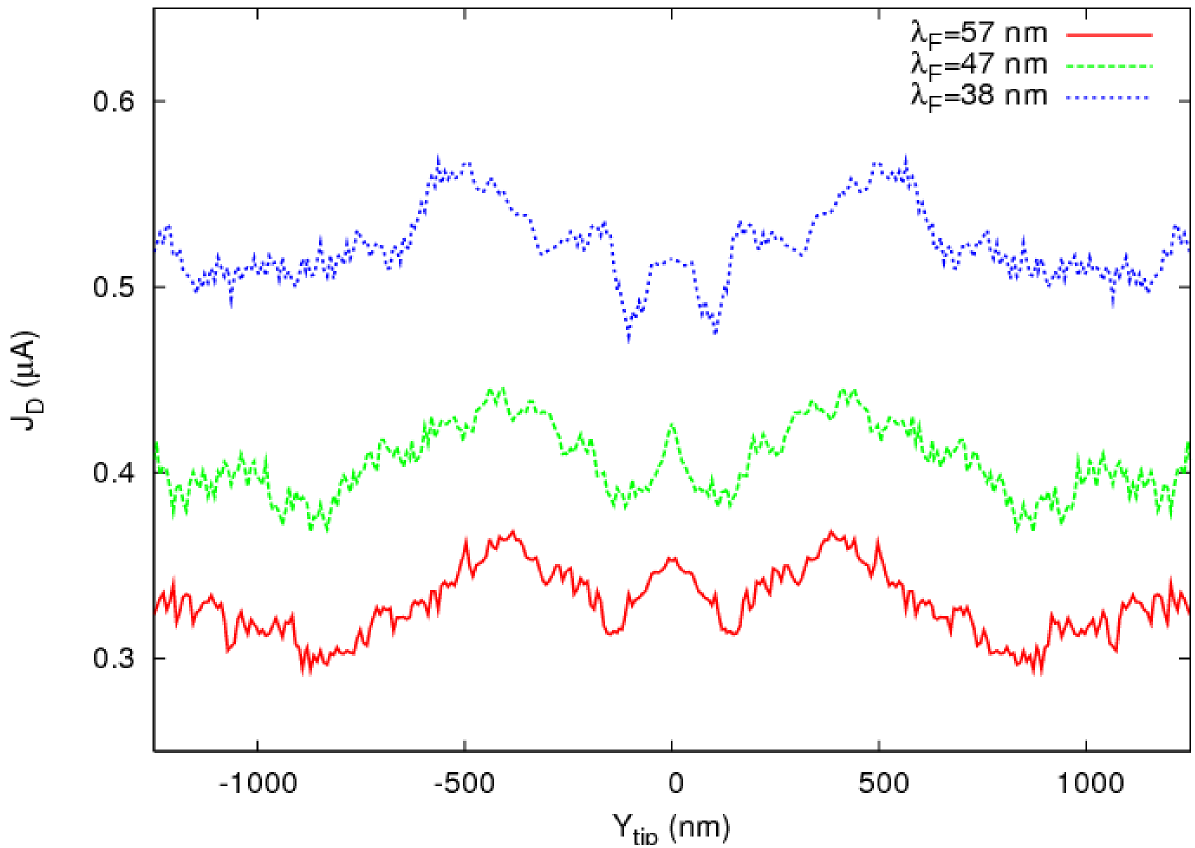
Simulation of SGM experiments



The current peak can be visualized in the current density maps only for the congested network

Simulation of SGM experiments

Uncongested network : influence of Fermi wavelength



A current peak is always visible when the tip is exactly above the central arm : larger transmission for a perfectly symmetric structure rather than a chaotic one

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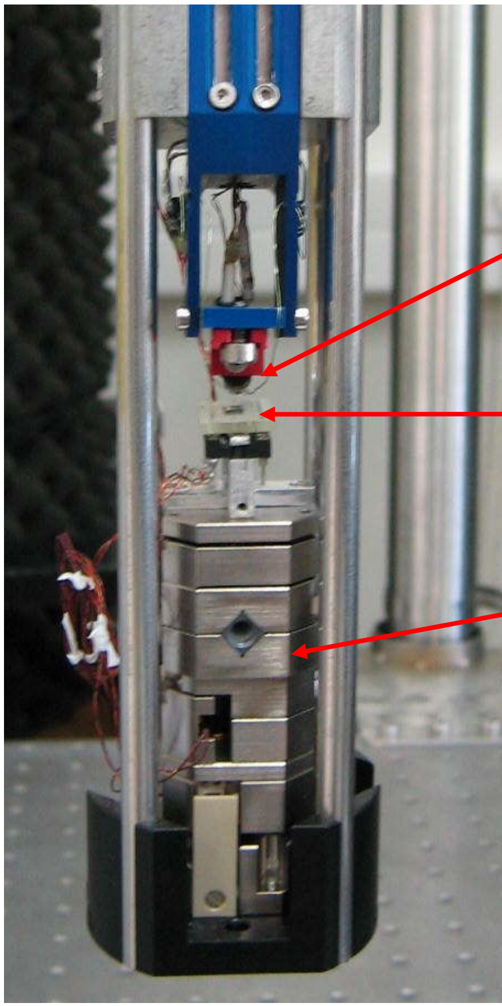
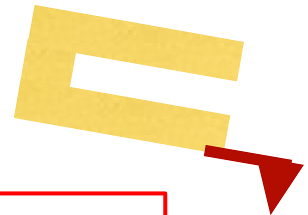
SGM experiments : microscope

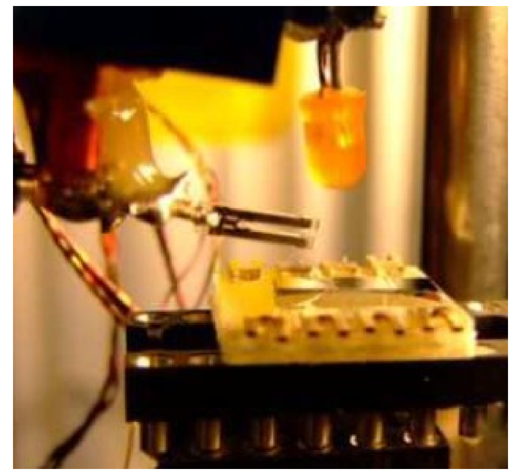
AFM @ 4 K and up to 11 T in Grenoble

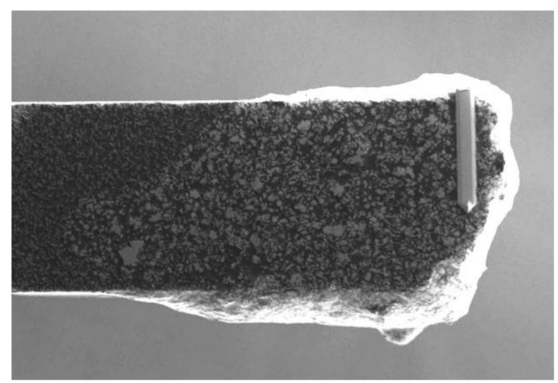
also available : 20 mK and up to 17 T in Louvain-la-Neuve

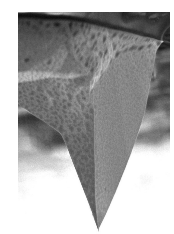




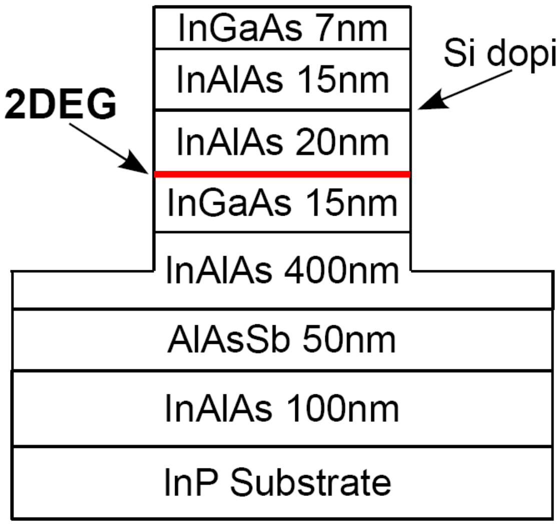
Probe = conductive AFM cantilever glued to a metallic pad of a tuning fork



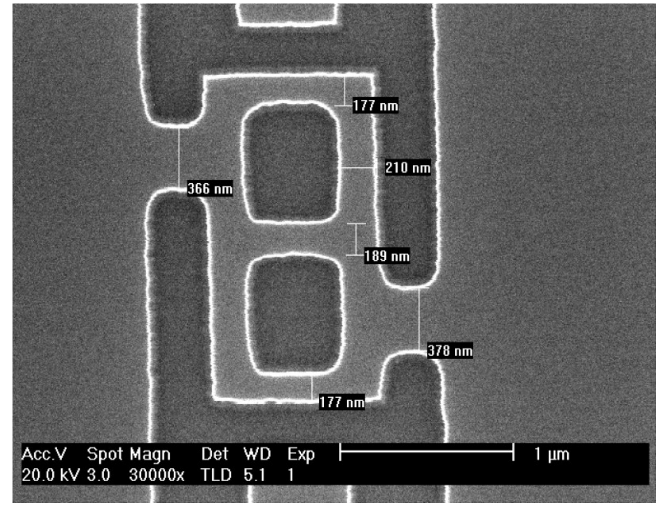




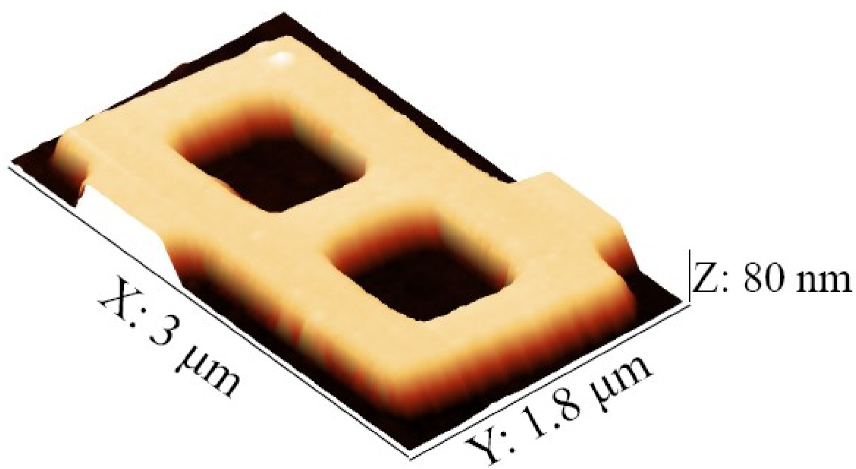
SGM experiments : device



<u>Collaborative work :</u>
- growth (IEMN)
- lithography (UCL)
- AFM+SGM (NEEL)

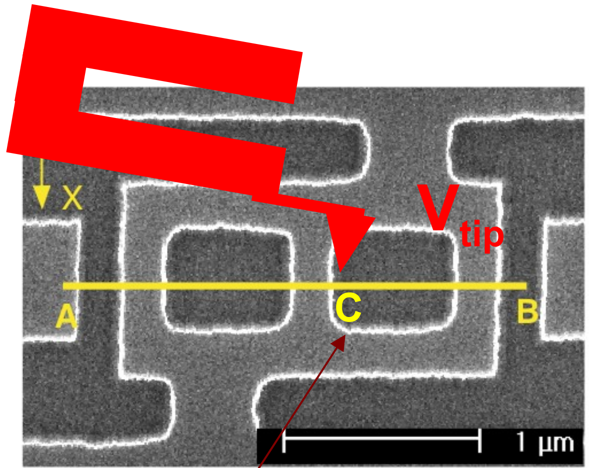


<u>2DEG properties :</u>
density = 3.5×10^{11} cm^{-2}
mobility = 100 000 cm^2/V/s
Fermi wavelength = 42 nm



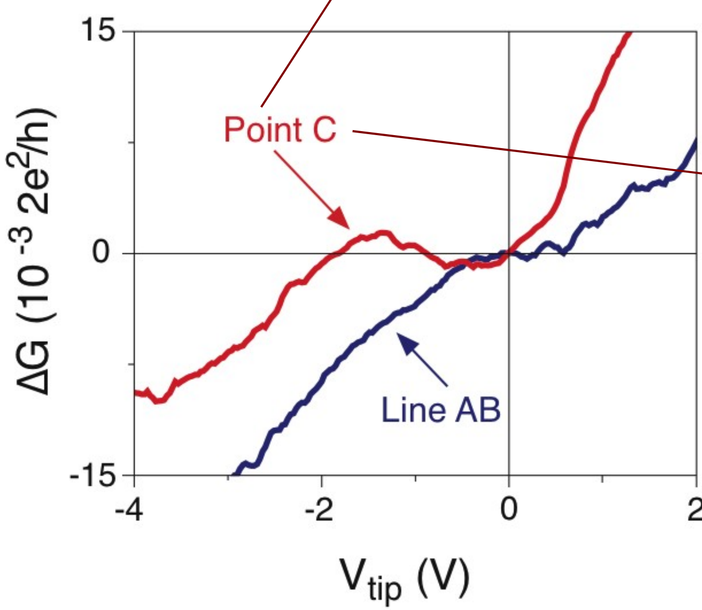
AFM topography at 4.2 K

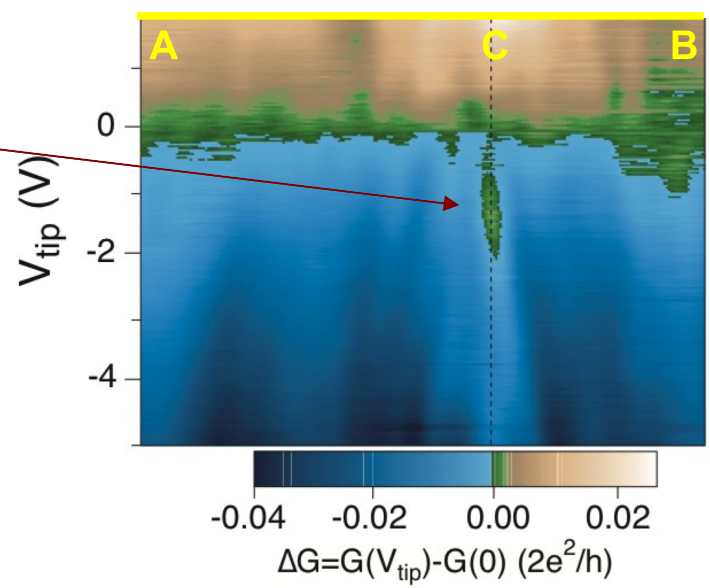
SGM experiments : Braess paradox



A small counter-intuitive conductance increase is observed around the central arm :

is it the same effect as the one observed in simulations ?





Conclusions

A Braess-like paradox has been evidenced in simulations of mesoscopic networks :

- *Closing a by-passing path can increase transport efficiency !*
- *Microscopic origin seems linked to current redistribution inside the network.*

Many open questions :

- *How far the analogy with the classical paradox can be made?*
- *What is the best suited geometry?*
- *Role of phase coherence and/or ballistic transport?*
- *Role of weak localization and interference?*
- *Influence of a magnetic field?*

More experiments are needed !

Thank you for your attention !