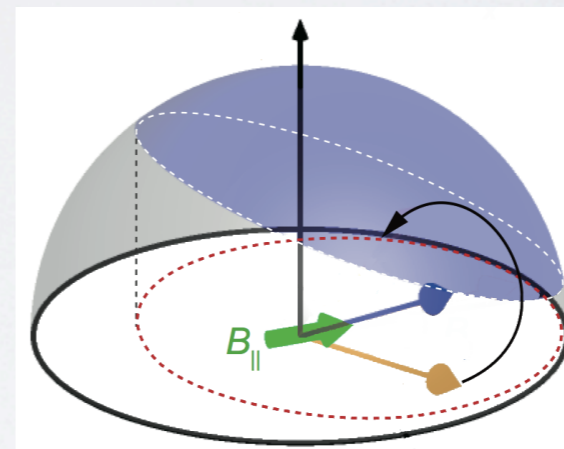
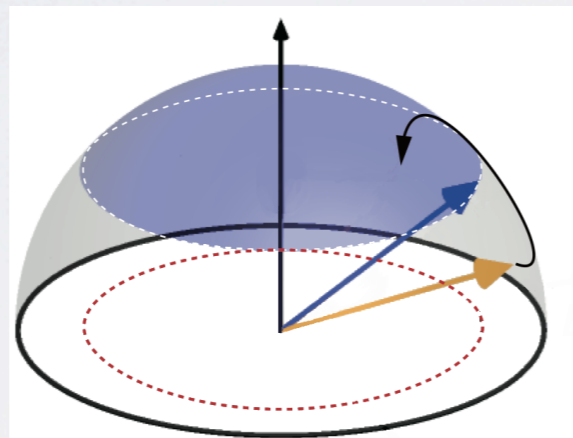


# Control of spin geometric phase in a semiconductor quantum ring



Henri Saarikoski, Regensburg University  
EP2DS/MSS Conference, Wrocław, 4th of July, 2013



F. Nagasawa, Diego Frustaglia, Henri Saarikoski,  
Klaus Richter, Makoto Kohda, and Junsaku Nitta (2013)



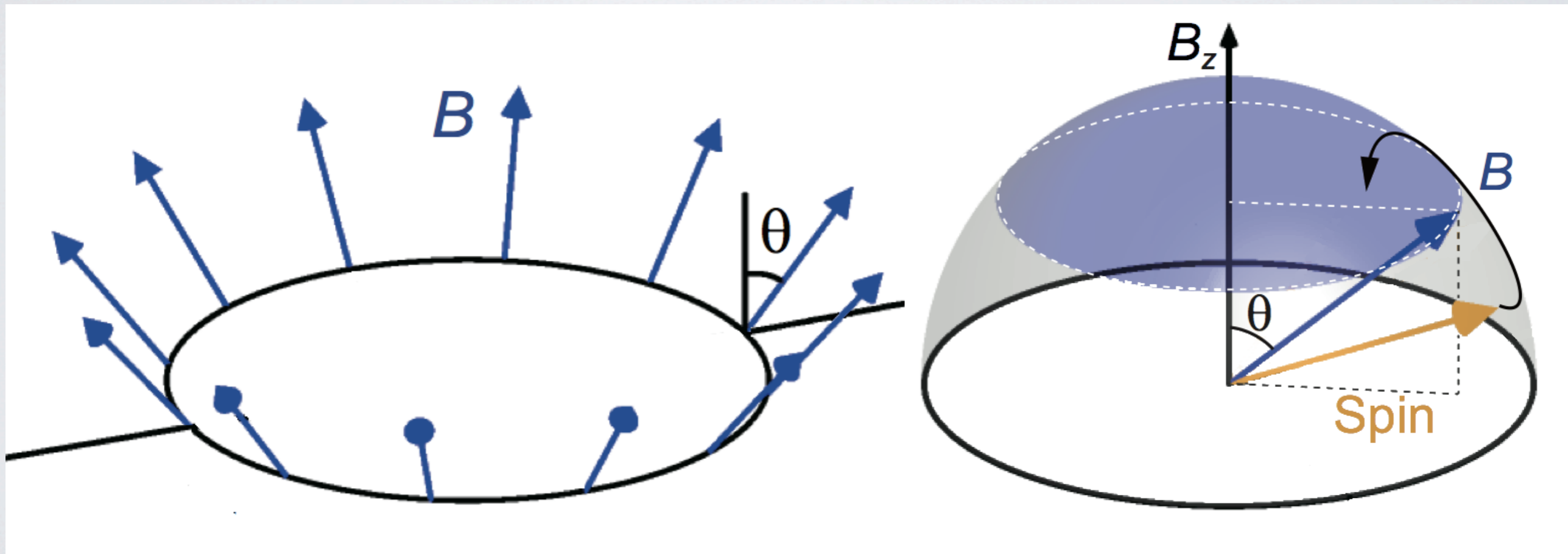
# Geometric phases

Geometric phases arise in wave systems where the parameters of the wave function are cycled around a circuit. (M. Berry, Proc. R. Soc. Lond. A 392, 45 (1984))

Can be observed via interference of waves traversing different paths.

Depends only on the geometry of the path :

robust against dephasing in contrast with the time-dependent dynamical phase.



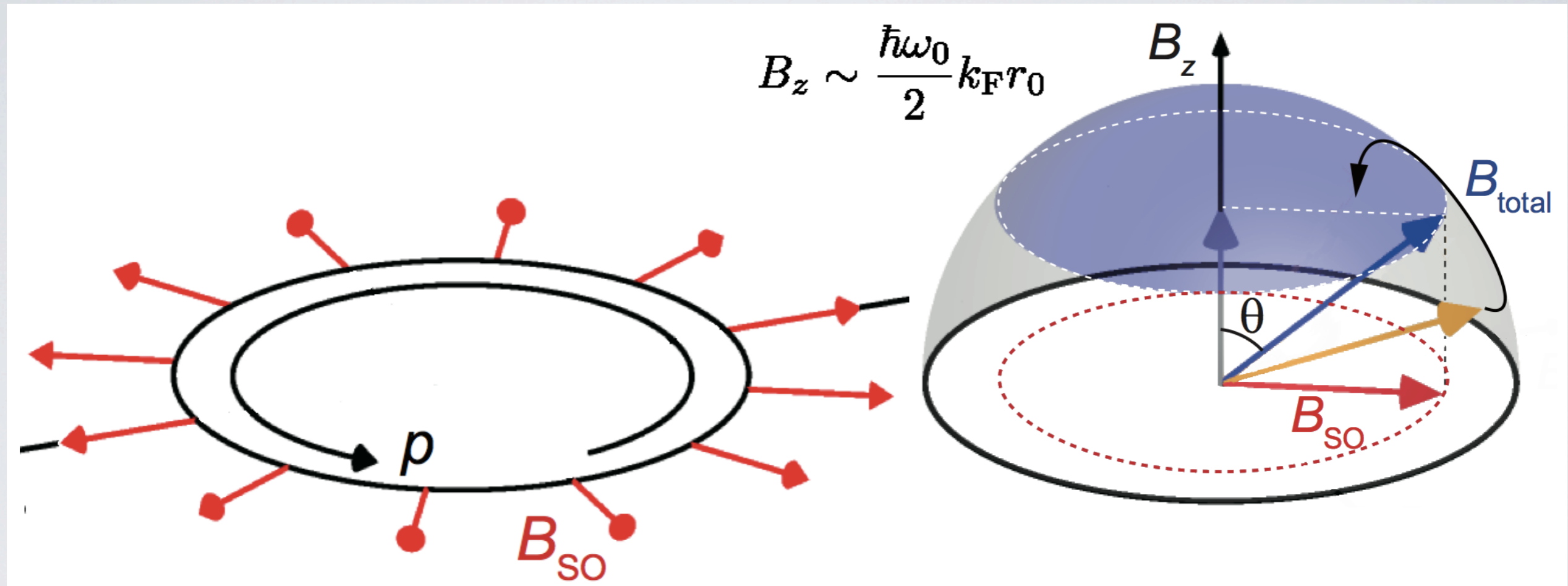
**Dynamic phase** : spin precession around  $B$

**Geometric phase** : solid angle subtended by spin eigenstates in  $B$

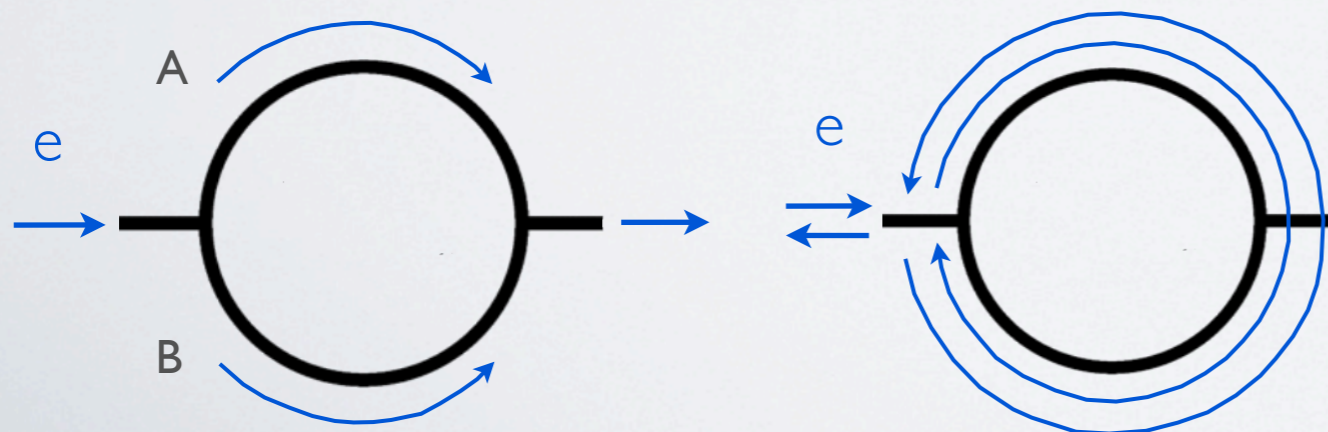
# Geometric phases in SO-coupled quantum rings

Extracted and studied for electron waves in InGaAs rings

F. Nagasawa et al., Phys. Rev. Lett. 108, 086801 (2012)

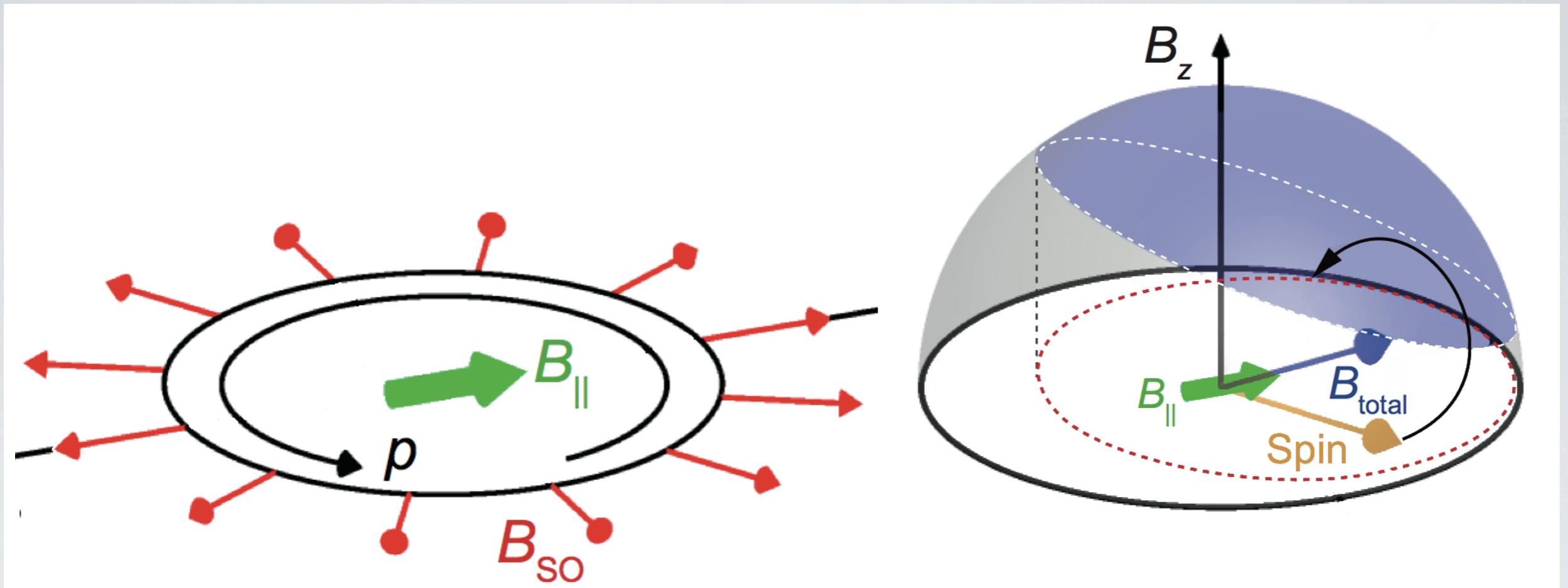


Aharonov-Anandan geometric phase in non-adiabatic evolution.



Interference of dynamic and geometric phases : oscillations in conductance,  
**Aharonov-Casher effect**

# Manipulation of the geometric phase



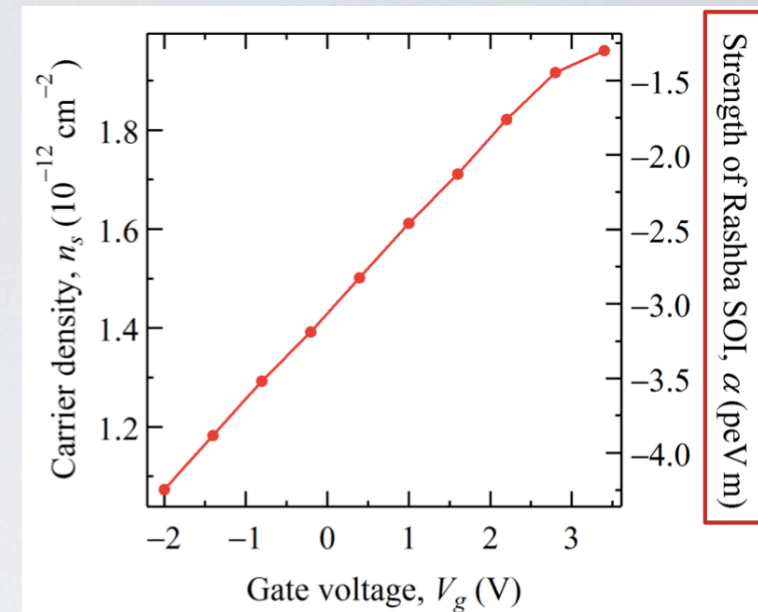
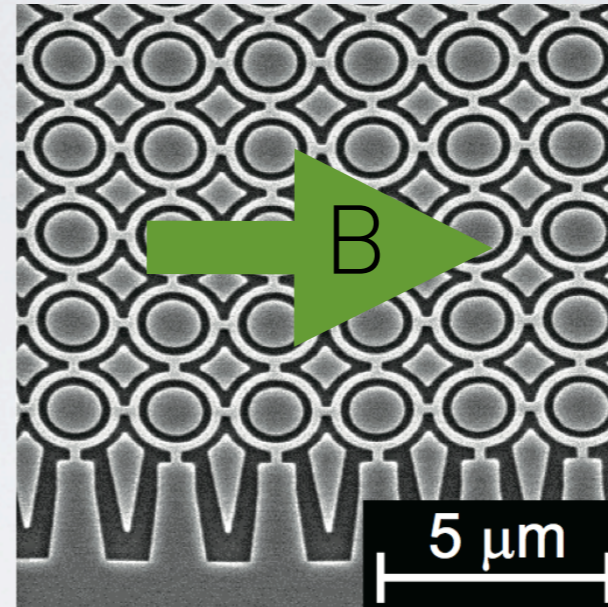
# Interference experiments in quantum rings

An array of 40 x 40 InGaAs/InAlAs rings  
multiple interference paths.

Multi-mode rings (~6 modes)

Tuning of the SO interaction with a gate.  
SdH analysis of coupling strength

$\omega_R \sim$  up to 20 GHz ,  $\omega_0 \sim$  6 GHz

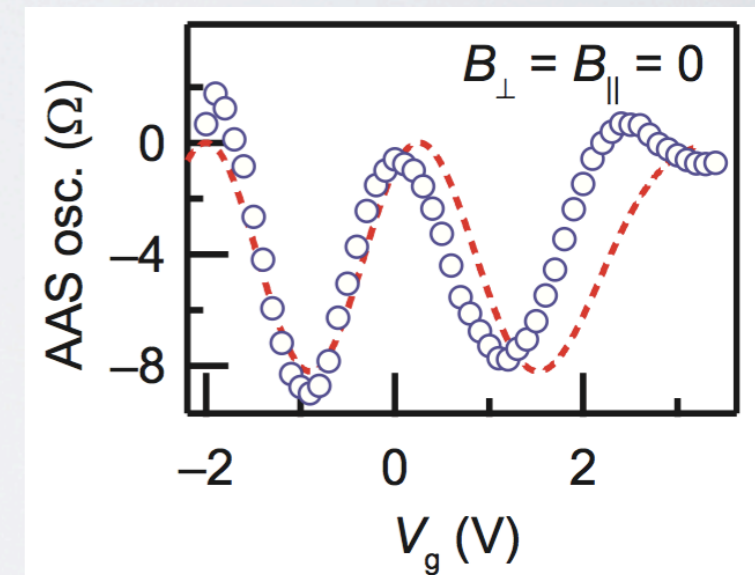
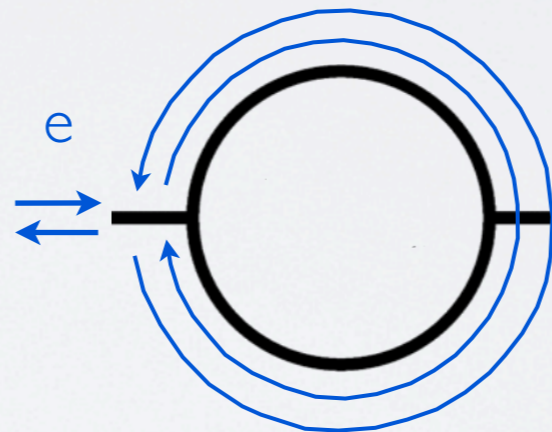


Quasi-ballistic rings.

Altshuler-Aronov-Spivak (AAS)

conductance oscillations :

interference in a full rotation around  
the ring in opposite directions



$$\frac{\delta R_{\alpha_R \neq 0}}{\delta R_{\alpha_R = 0}} \propto \cos \left[ 2\pi \left( \sqrt{1 + Q_R^2} - 1 \right) \right] = \underbrace{\cos [2\pi Q_R \sin \theta]}_{\text{dynamic}} \underbrace{- 2\pi(1 - \cos \theta)}_{\text{geometric}}$$

# Geometric phase shift in low Zeeman fields

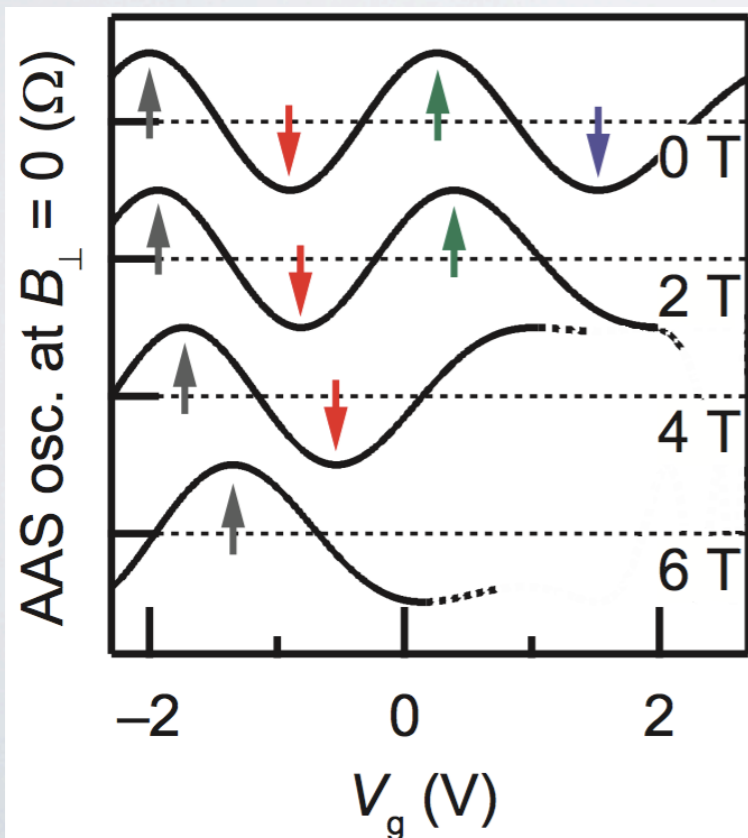
Phase shift, 1st order perturbation theory (D. Frustaglia) :

$$\phi = \left( \frac{\omega_B}{\omega_0 k_F r} \right)^2 \frac{4 + Q_R^2}{4Q_R^2 \sqrt{1 + Q_R^2}}$$

$$\begin{aligned} 2\pi\phi_{AA} &= 2i \int_0^{2\pi} d\varphi \langle \overline{n, \lambda, s} | \frac{\partial}{\partial \varphi} | \overline{n, \lambda, s} \rangle \\ &= -2\pi[(1 - \cos \theta) - \phi - 2j] \end{aligned}$$

Pure geometric phase modulation. Independent of dynamical phases

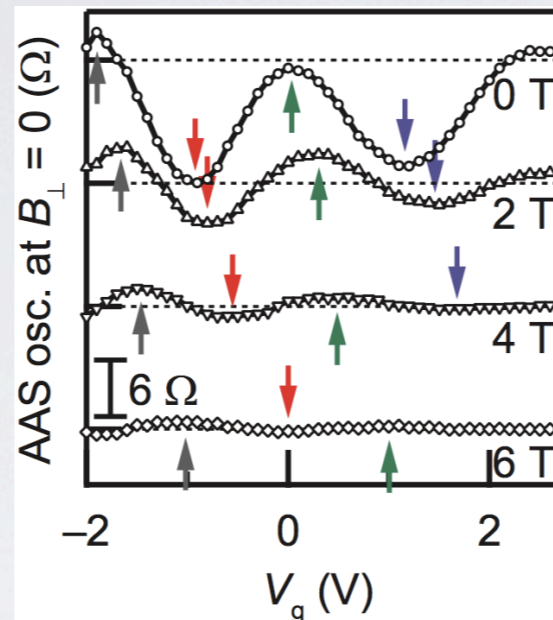
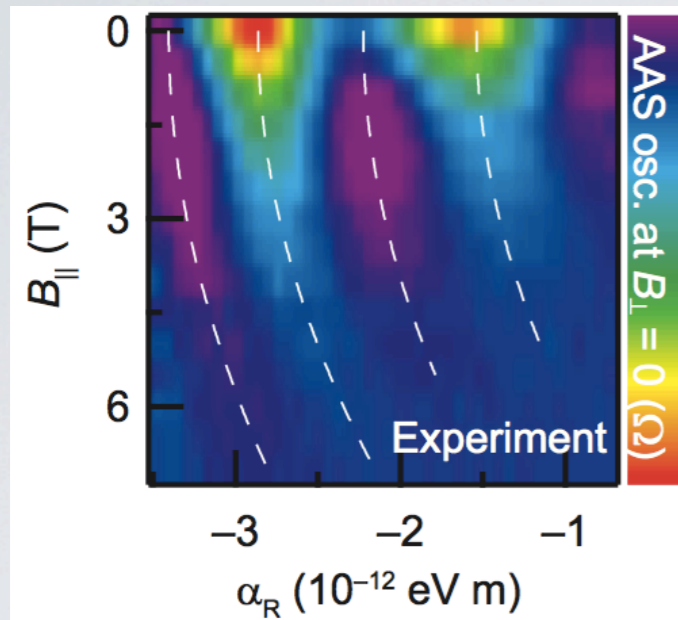
$$G \approx \frac{e^2}{h} \left\{ 1 + \cos \left[ \pi \left( \sqrt{1 + Q_R^2} - 1 + \left( \frac{\omega_B}{\omega_0 k_F r_0} \right)^2 \frac{4 + Q_R^2}{4Q_R^2 \sqrt{1 + Q_R^2}} \right) \right] \right\}$$



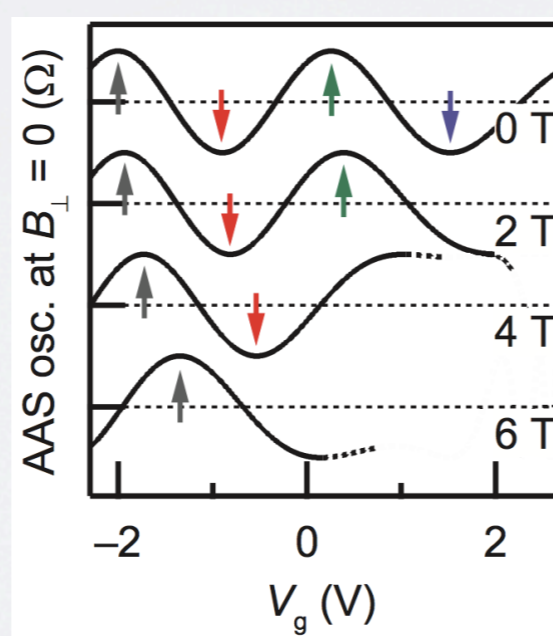
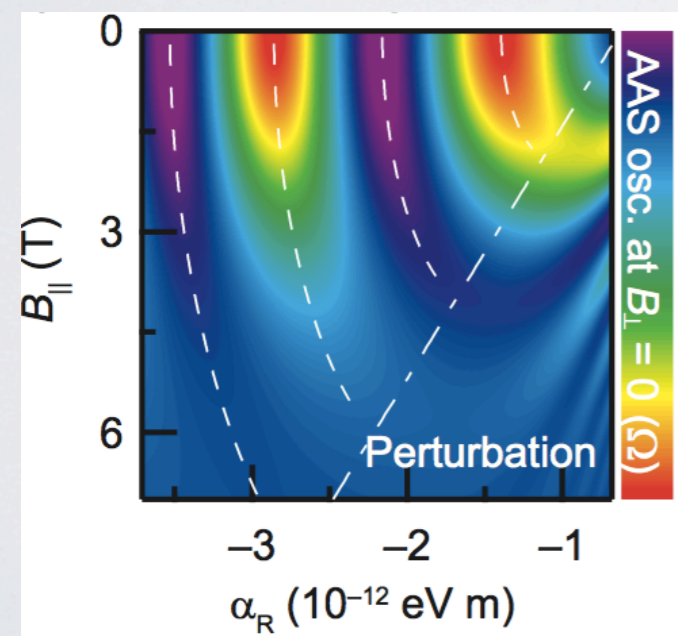
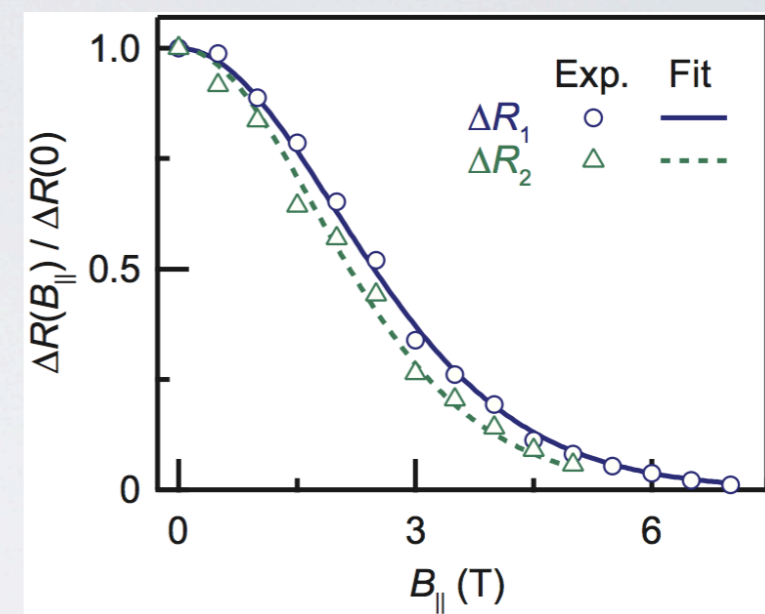
Calculated also for weakly coupled rings F. K. Joibari, Y. M. Blanter & G. E. W. Bauer, Aharonov-Casher effect in quantum ring ensembles. arXiv:1304.6195 [cond-mat.mes-hall] (2013).

# Dip and peak shifts of AAS oscillations

Quadratic phase shift, decoherence with B



TR symmetry breaking:  
suppression of spin interference



$$\frac{\Delta R(B_{\parallel})}{\Delta R(0)} = \exp \left[ -4\pi r \left( \frac{1}{l_{\varphi}(B_{\parallel})} - \frac{1}{l_{\varphi}(0)} \right) \right]$$

F. E. Meijer, A. F. Morpurgo, T. M. Klapwijk,  
and J. Nitta, PRL 94, 186805 (2005)

# Computational approach

Multiple transport modes, disorder and high Zeeman fields

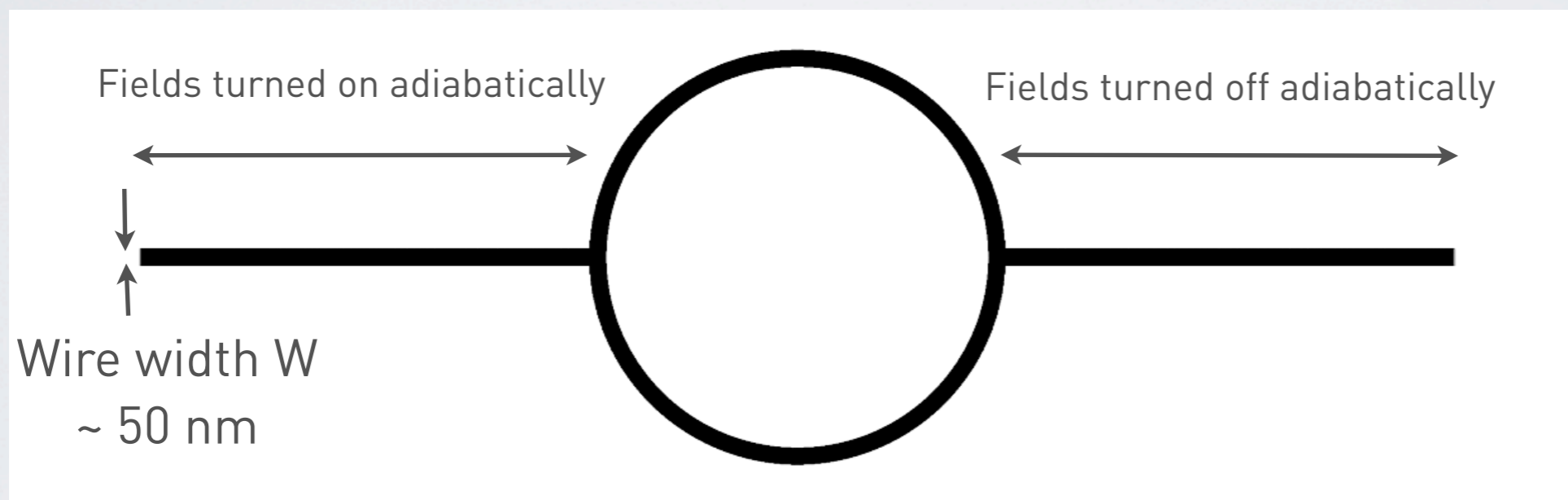
$$\hat{H} = \frac{1}{2m^*} \hat{\mathbf{P}}^2 + \alpha(k_y\sigma_x - k_x\sigma_y) + \frac{1}{2}g_{\text{eff}}\mu_B B\sigma_x + V_{\text{dis}}(\mathbf{x})$$

Bychov-Rashba SOI

Zeeman field

Anderson-like  
disorder potential

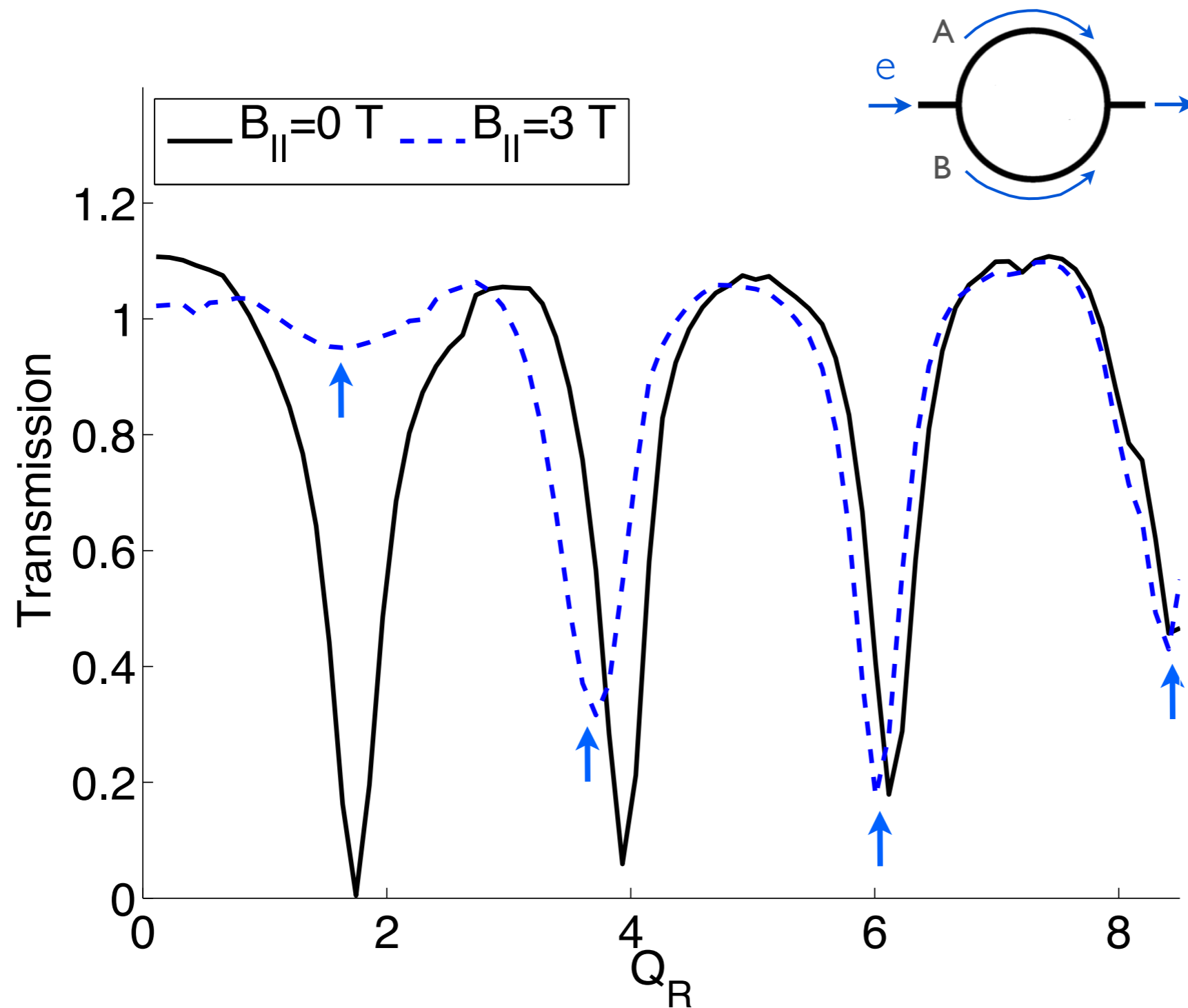
Transport equation solution with the Recursive Green's Function method (RGFM)



Mean free path in the numerical calculations = 2-6 microns,  $T = 1.7$  K,  $g$ -factor = 3, ring displacement 10-20 nm, no Dresselhaus term

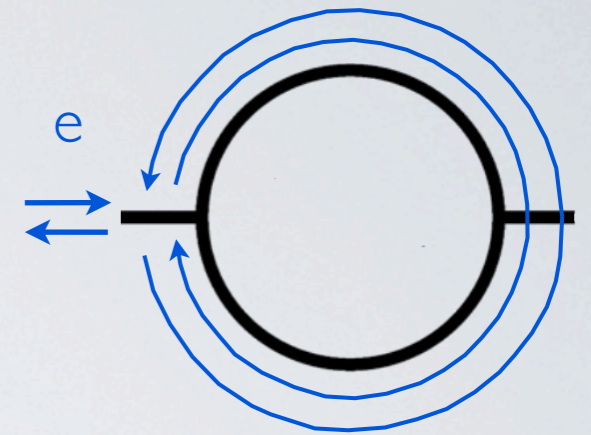
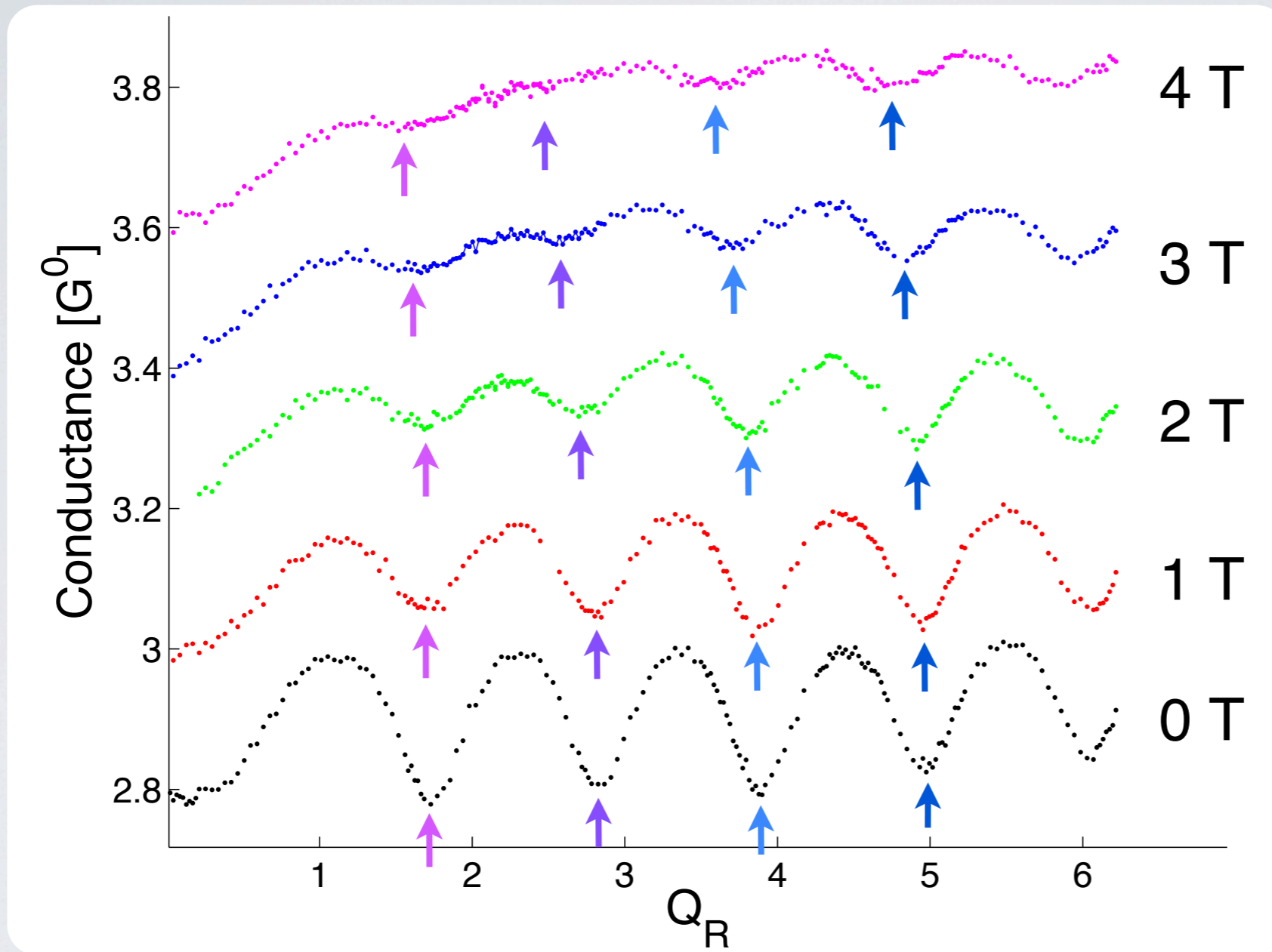
# RQMG method in ballistic rings

Geometric phase shift for direct interference paths through a ballistic ring.



# AAS oscillations in quasi-ballistic multi-mode rings

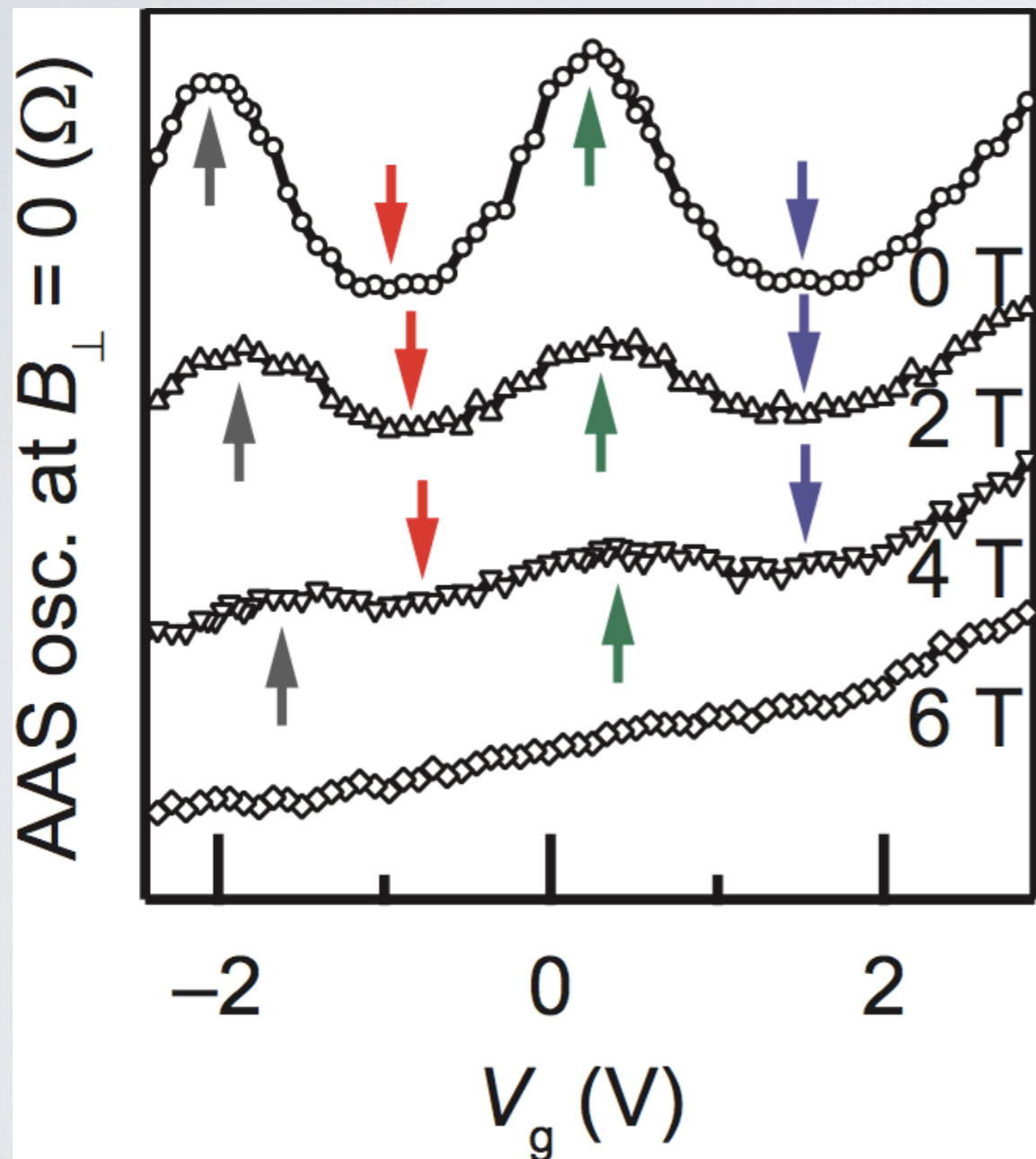
$r_0 = 608$  nm ring, mean free path =  $2 \mu\text{m}$



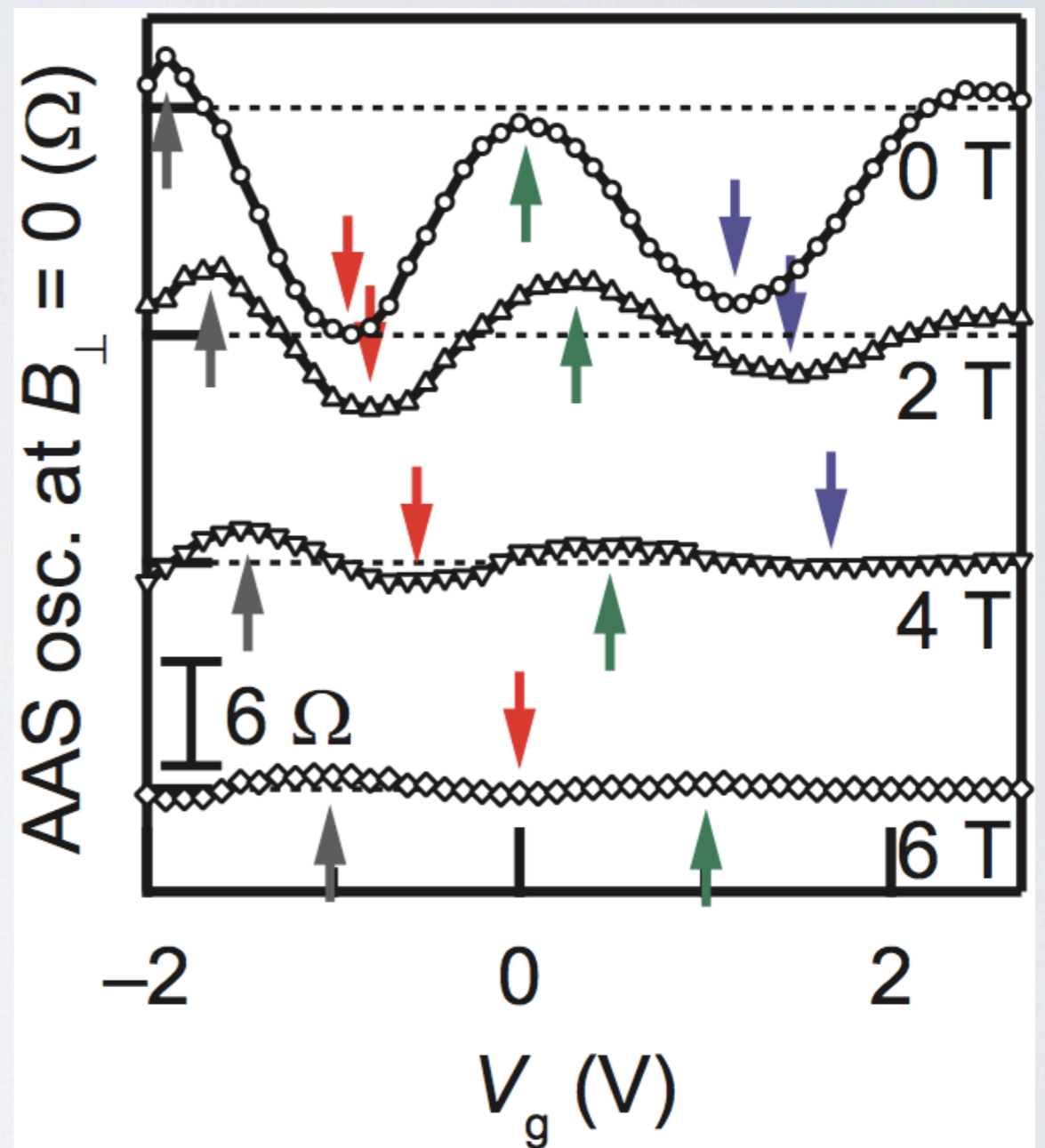
Dips shifts to lower SO fields; an additional phase shift due to the magnetic field.

# Simulations of AAS conductance oscillations

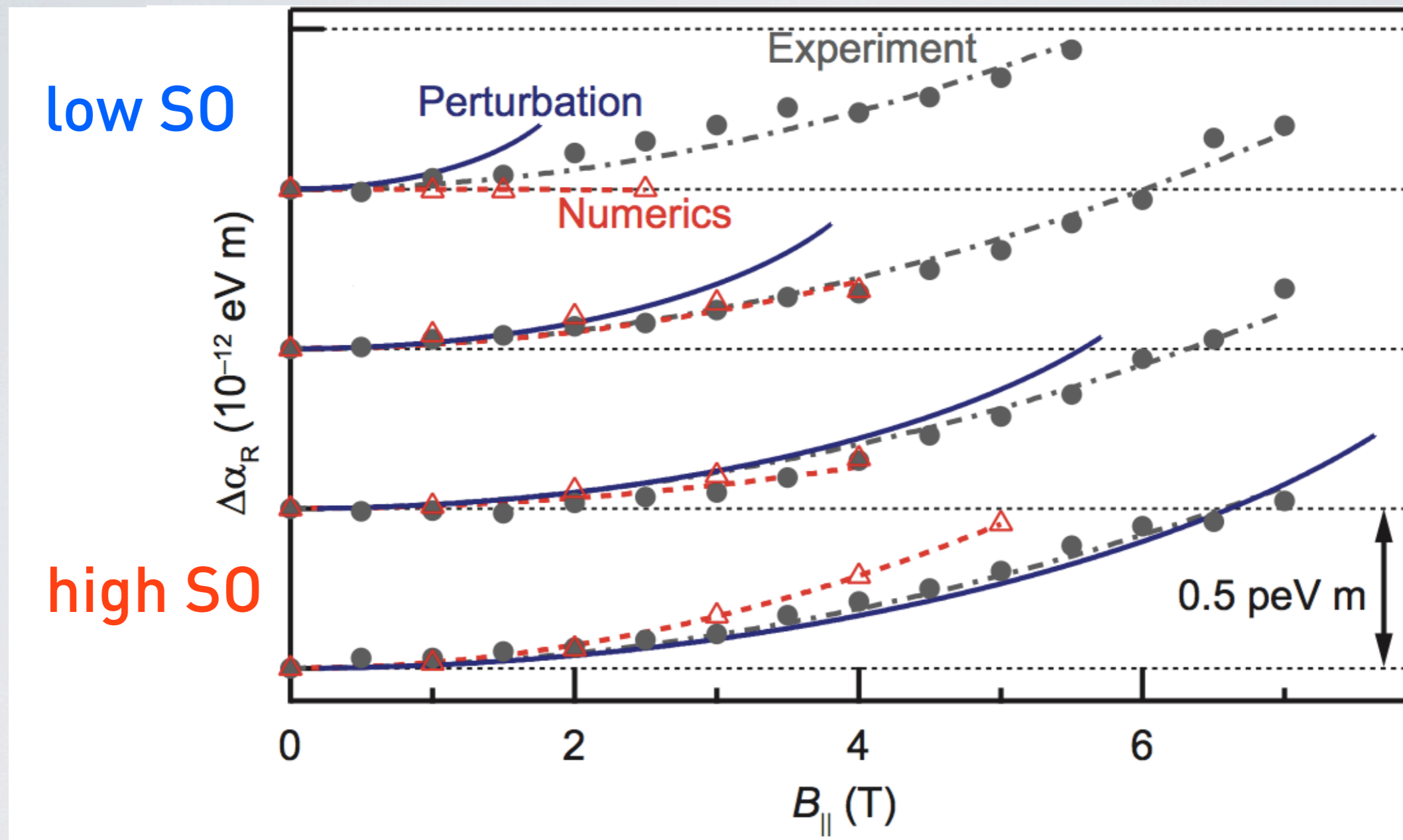
Numerical RGFM



Experiments



# Phase shifts : experiments vs. theory



Quadratic shift calculated and observed in experiments

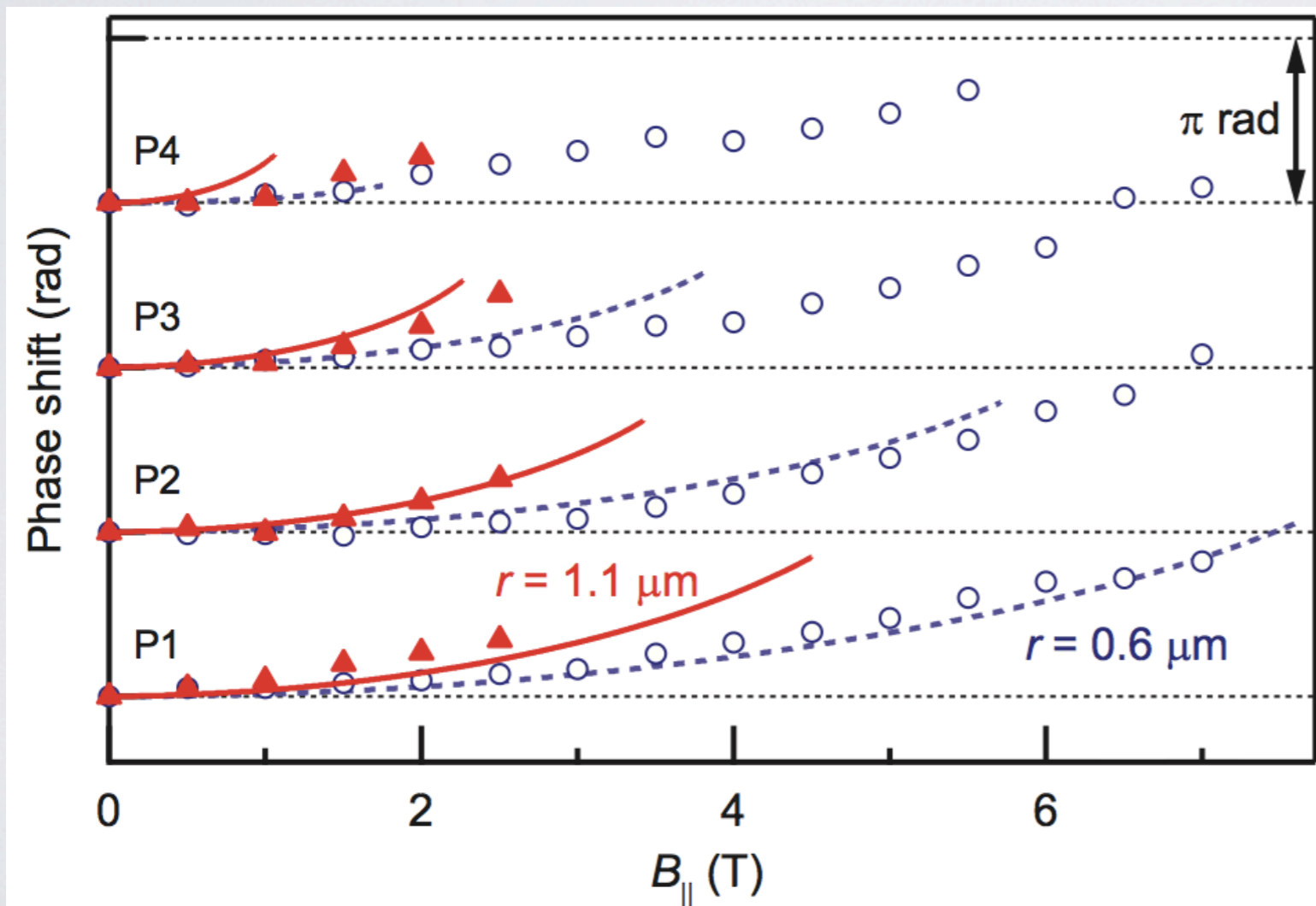
Perturbation theory works fine in high SO fields :  
phase shift is dominated by a few modes or just the lowest one due to dephasing.

# Radius dependence

$$\phi = \left( \frac{\omega_B}{\omega_0 k_F r} \right)^2 \frac{4 + Q_R^2}{4Q_R^2 \sqrt{1 + Q_R^2}}$$

$$\omega_0 = \hbar / (m^* r^2)$$

$$\phi \sim r^2$$



Qualitative agreement with the perturbation theory

# Summary

Geometric phase shift with the in-plane magnetic field:  
quadratic in the in-plane magnetic field strength

Phase manipulation independent of the dynamic phase and  
without resorting to other geometric phases such as the Aharonov-Bohm phase

Phase shift dominated by a few modes or just the lowest one due to dephasing

## Collaboration

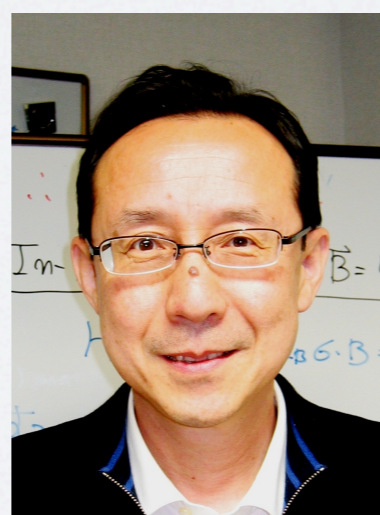
Tohoku group (experiments)



Fumiya Nagasawa



Makoto Kohda



Junsaku Nitta



Diego Frustaglia



Klaus Richter

Sevilla

Regensburg