Quantum theory of the second-order electromagnetic response of a low-dimensional electron gas

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The linear electromagnetic response of a uniform electron gas to a potential electric field is determined by the known Lindhard formula for the dielectric function $\varepsilon(q,\omega)$. This formula can be obtained within the self-consistent-field approach or in the random phase approximation. The function $\varepsilon(q,\omega)$ is related to the polarizability of the electron gas $\alpha(q,\omega)$ and can be analytically calculated for three-, two- and one-dimensional (3D, 2D, 1D) degenerate electron gases at arbitrary values of the wave vector q and the frequency ω.

In this work we derive simple analytical formulas which relate the *second-order* response functions, i.e. the second polarizability $\alpha^{(2)}(q,\omega)$ and the corresponding to $\varepsilon(q,\omega)$ second-order function, to the first-order (linear) ones. These formulas are valid for 3D, 2D and 1D electron gases obeying both Boltzmann and Fermi statistics (i.e. for an arbitrary relation between the chemical potential and temperature). For a degenerate *n*D electron gas (n=1,2,3) they allow one to analytically calculate the second-order response functions at arbitrary values of q and ω.

The calculated second-order response functions describe the second harmonic generation in the considered electron systems. It has been recently shown [1] that the excitation of the 2D electron gas by the electromagnetic wave with the frequency ω close to the 2D plasma frequency $\omega_p(q)$ should lead to a huge enhancement of the second harmonic intensity. The results of [1] were obtained in the long-wavelength and low-frequency limit (q and ω are small as compared to the Fermi wave vector and energy respectively). The new formulas obtained in this work are valid at arbitrary q and ω and allow one to get the optimal conditions for the second harmonic generation not restricted by the long-wavelength and low-frequency limits. They are valid both for quantum-well (2D) and for quantum wire (1D) systems.

This work is very important for development of semiconductor sources of sub-terahertz and terahertz radiation. The experimental technique of exciting the 2D and 1D plasmons in semiconductor GaAs/AlGaAs structures is very well developed and the mobility of GaAs samples is so high that the quality factor of plasma resonances $Q=\omega_p/\delta\omega_p$ can be much larger than unity, Q>>1. Since the intensity of the second harmonic was shown [1] to be proportional to Q^4 one should expect a very efficient frequency up-conversion at the 2D/1D plasmon frequencies. Our results may thus open a new research direction in fundamental and applied physics – the nonlinear plasmonics.

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[1] S. A. Mikhailov, Phys. Rev. **84,** 045432 (2011).